7-1995

Labor Scheduling Using NPV Estimates of the Marginal Benefit of Additional Labor Capacity

Gary Thompson
Cornell University School of Hotel Administration, gmt1@cornell.edu

Labor Scheduling Using NPV Estimates of the Marginal Benefit of Additional Labor Capacity

Abstract
An extensive literature exists on the problems of daily (shift) and weekly (tour) labor scheduling. In representing requirements for employees in these problems, researchers have used formulations based either on the model of Dantzig (1954) or on the model of Keith (1979). We show that both formulations have weakness in environments where management knows, or can attempt to identify, how different levels of customer service affect profits. These weaknesses results in lower-than-necessary profits.

This paper presents a New Formulation of the daily and weekly Labor Scheduling Problems (NFLSP) designed to overcome the limitations of earlier models. NFLSP incorporates information on how changing the number of employees working in each planning period affects profits. NFLP uses this information during the development of the schedule to identify the number of employees who, ideally, should be working in each period. In an extensive simulation of 1,152 service environments, NFLSP outperformed the formulations of Dantzig (1954) and Keith (1979) at a level of significance of 0.001. Assuming year-round operations and an hourly wage, including benefits, of $6.00, NFLSP's schedules were $96,046 (2.2%) and $24,648 (0.6%) more profitable, on average, than schedules developed using the formulations of Danzig (1954) and Keith (1979), respectively. Although the average percentage gain over Keith's model was fairly small, it could be much larger in some real cases with different parameters. In 73 and 100 percent of the cases we simulated NFLSP yielded a higher profit than the models of Keith (1979) and Danzig (1954), respectively.

Keywords
service operations, personnel and shift scheduling, staffing

Disciplines
Business Administration, Management, and Operations

Comments

Required Publisher Statement

Labor Scheduling Using NPV Estimates of the Marginal Benefit of Additional Labor Capacity

Gary M. Thompson

David Eccles School of Business, University of Utah

Journal of Operations Management (1995), 13, 67-86.

Author Note:

Address correspondence to Gary M. Thompson, David Eccles School of Business, University of Utah, Salt Lake City, UT 84112

The author wishes to thank the anonymous referees for their helpful comments. This research was supported in part by an Eccles Summer Research Fellowship.

Abstract

An extensive literature exists on the problems of daily (shift) and weekly (tour) labor scheduling. In representing requirements for employees in these problems, researchers have used formulations based either on the model of Dantzig (1954) or on the model cf Keith (1979). We show that both formulations have weaknesses in environments where management knows, or can attempt to identify, how different levels of customer service affect profits. These weaknesses result in lower-than-necessary profits.

This paper presents a New Formulation of the daily and weekly Labor Scheduling Problems (NFLSP) designed to overcome the limitations of earlier models. NFLSP incorporates information on how changing the number of employees working in each planning period affects profits. NFLSP uses this information during the development of the schedule to identify the number of employees who, ideally, should be working in each period. In an extensive simulation of 1,152 service environments, NFLSP outperformed the formulations of Dantzig (1954) and Keith (1979) at a level of significance of 0.001. Assuming year-round operations and an hourly wage, including benefits, of $6.00, NFLSP’s schedules were $96,046 (2.2%) and $24,648 (0.6%) more profitable, on average, than schedules developed using the formulations of Dantzig (1954) and Keith (1979), respectively. Although the average percentage gain over Keith’s model was fairly small, it could be much larger in some real cases with different parameters. In 73 and 100 percent of the cases we simulated NFLSP yielded a higher profit than the models of Keith (1979) and Dantzig (1954), respectively.

Keywords: Service operations; Personnel and shift scheduling; Staffing

1. Introduction

Labor scheduling is a key function performed by managers of service delivery systems. Four separate tasks, which we shall identify as Tl, T2, T3 and T4, comprise this function (Thompson, 1993). In these tasks, a manager forecasts customer demand for the service (Tl), translates the forecasts of customer demand into requirements for employees (T2), develops the cheapest labor schedule that always has the correct number of employees present (T3), and controls the delivery of the service in real-time (T4). The scope of labor scheduling has typically been either a day (the shift scheduling problem) or a week (the tour scheduling problem). Researchers have focused primarily upon T3 in the extensive literature on the problems of daily and weekly labor scheduling. Mathematical programming models developed for T3 have used formulations based on the model of Dantzig (1954) or on the model of Keith (1979).
These models treat employee requirements as minimum acceptable levels and as target levels, respectively. Unfortunately, both formulations have weaknesses in environments where management has some knowledge of how various levels of customer service affect profits, as we will address later.

This paper develops a new formulation of the daily and weekly labor scheduling problems for cases where management knows, but not necessarily with certainty, the cost of poor and the benefit of good customer service. We show the superiority of the new formulation via an experiment simulating 1,152 service environments. The structure of the remainder of the paper is as follows. Section 2 presents an overview of relevant labor scheduling research; Section 3 introduces the new formulation of the daily and weekly labor scheduling problems; Section 4 describes the design and Section 5 presents the results of a model validation experiment; and Section 6 concludes the paper with a discussion of the findings.

2. Labor scheduling background

This section first presents key historical formulations of the daily and weekly labor scheduling problems and then identifies limitations of these formulations. Throughout the remainder of the paper we shall refer to the tour scheduling problem, although our comments also apply to the shift scheduling problem.

2.1. Key historical formulations of the daily and weekly labor scheduling problems

Dantzig (1954) developed the first mathematical programming formulation of the daily and weekly labor scheduling problems. His model, DFLSP, may be represented as

$$\min Z = \sum_{t \epsilon T} c_t x_t$$

subject to

$$\sum_{t \epsilon T} a_{tp} x_t \geq r_t \qquad \text{for } p \epsilon P$$

$$x_t \geq 0 \text{ and integer} \qquad \text{for } t \epsilon T$$

Here T is the set of unique tours that can be scheduled; c_t is the cost of assigning an employee to tour t; x_t is the number of employees working tour t; $a_{tp} = 1$, if period p is a working period for tour t, $a_{tp} = 0$, otherwise; P is the set of planning intervals in the weekly operating horizon; and r_p is the number of employees management wishes to have working in period p.

In DFLSP, the objective (1) is to minimize the total cost of the scheduled tours. Constraint set (2) ensures that no fewer than the desired number of employees work in every period of the week. Constraint set (3) specifies the integrality of the decision variables.

Keith (1979) formulated the daily and weekly labor scheduling problems to allow for both under- and overstaffing. His model, which we shall refer to as KFLSP, used two understaffing and two overstaffing variables for each planning period to measure deviations from the desired staff size. Because one under- and one overstaffing variable from each period are bounded, and because these bounded variables have lower costs than the corresponding unbounded variables, KFLSP's solutions tend to have employee shortages and surpluses distributed within the specified bounds. KFLSP is

$$\min Z = \sum_{t \epsilon T} c_t x_t + \sum_{p \epsilon P} \left[10\alpha_p + 3\beta_p + 0\sigma_p + 0.5\pi_p\right]$$

subject to

$$\sum_{t \epsilon T} a_{tp} x_t + \alpha_p + \beta_p - \sigma_p - \pi_p = r_p \qquad \text{for } p \epsilon P$$

$$\beta_p \leq b_p^{\beta} \qquad \text{for } p \epsilon P$$

$$\sigma_p \leq b_p^{\sigma} \qquad \text{for } p \epsilon P$$

$$\alpha_p \leq r_p - b_p^{\beta} - 1 \qquad \text{for } p \epsilon P$$

$$\alpha_p, \beta_p, \sigma_p, \pi_p \geq 0 \text{ and integer} \qquad \text{for } p \epsilon P$$

$$x_t \geq 0 \text{ and integer} \qquad \text{for } t \epsilon T$$

Here α_p is the variable representing the unbounded shortage of employees in period p, measured in employee-periods; β_p is the variable representing the bounded shortage of employees in period p, measured in employee-periods; σ_p is the variable representing the bounded surplus of employees in period p, measured in employee-periods; $\pi_p$is the variable representing the unbounded surplus of employees in period p, measured in employee-periods; b_p^β is the limit on the bounded shortage of employees in period p, measured in employee-periods; and b_p^σ is the limit on the bounded surplus of employees in period p, measured in employee-periods.

KFLSP's objective (4) is to find the schedule with the lowest total cost. Relevant costs include the costs of having employees work specific tours and the artificial costs of employee shortages or surpluses. The artificial costs of the employee shortages and surpluses used in our investigation and shown in Eq. (4), although arbitrary, are consistent with the suggestions of Keith (1979).

For each planning period, constraint set (5) ensures that the number of employees working equals the desired staff size, adjusted by any employee shortage or surplus. Constraint sets (6) and (7) impose bounds on the variables measuring the bounded shortage and surplus of employees in each period, respectively. We imposed the limits on the bounded under- and overstaffing variables as follows:

$$b_p^\beta = b_p^\sigma = \min\left(3, \max\left(1, \lfloor 0.2 r_p + 0.5 \rfloor\right)\right)$$

where $\lfloor y \rfloor$ is the largest integer $\leq y$.

Although Keiths' model (Keith, 1979) did not include constraint set (8), we include it to ensure that at least one employee works each period. Finally, constraint sets (9) and (10) specify the integrality of the employee shortage and surplus variables and tour variables, respectively.

2.2. Limitations of current formulations of the daily and weekly labor scheduling problems

Crucial inputs to both DFLSP and KFLSP are each planning period's desired number of employees (the r_p). In determining the desired number of employees for a period, some researchers have used a customer service policy of the form "serve at least x percent of customers within time y" (Buffa et al., 1976; Segal, 1974). A reasonable question is: Why might such a service policy exist? The answer is that management believes that their organization suffers—by losing current or future sales—when the facility provides a poorer level of customer service. A service policy, then, is but a surrogate measure of the true effect of different levels of service on profits. Davis (1991) presented a methodology for managers wishing to measure how different service levels affect long-run profits. Quinn et al. (1991) reported a substantial increase in profit at L.L. Bean after the organization switched to a profit orientation to providing service from the existing service-level orientation. It thus behooves labor scheduling researchers to use the more accurate information on the costs and benefits of different service levels provided by Davis (1991) and Quinn et al.'s (1991) methodologies. Hence, we arrive at the environment for this study: situations where managers know, or can attempt to identify, how various levels of customer service affect long-run profits.

Now that we have identified the study environment, consider how to set the desired staffing levels in DFLSP and KFLSP. Upon initial thought, one would likely choose the staffing level that maximizes the total net-present-value (NPV) profit for the period.[1] This poses problems for situations where the per-period tour costs both are and are not identical. In the former case, setting the desired staffing levels in the stated manner only makes sense if one can exactly satisfy these staffing levels in all periods. If one cannot exactly satisfy all desired staffing levels, total profits will likely increase if one lowers the desired staffing levels in some periods.

[1] The rationale for using NPV is that the true impact on the organization of providing poor service (or good service, for that matter) is unlikely to show up in the current planning horizon, *particularly* given the single week horizon of the labor tour scheduling problem.

This is because, *overall*, a tour may have a greater cost than the benefit accruing from the improved customer service that it provides. In the latter case (unequal tour costs, per period), the particular tours that contribute staff in a period, *which one cannot divine before developing the labor schedule*, affect the period's total relevant NPV profit. Limitation 1 shall refer to the difficulty of appropriately setting the desired staffing levels used as inputs to DFLSP and KFLSP's in the environments we consider.

A second limitation of DFLSP, Limitation 2, is that it assumes surplus employees are of equal value across all periods. Limitation 2 also applies to KFLSP: although KFLSP uses two-tiered costs for under- and overstaffing, the costs for each tier are equal within and across periods. This limitation does not become apparent until one examines the nature of service delivery systems. Typically, one can best describe organizations' service delivery functions as queuing systems. A well-known characteristic of queuing systems is the diminishing influence on system performance of increasing the number of service delivery personnel. For example, adding two employees in a period improves customer service by less than twice the improvement in customer service that occurs when adding only a single employee.

Table 1 categorizes germane daily and weekly labor scheduling literature. Unfortunately, Limitation 1 and Limitation 2 plague all the applicable literature, as Table 1 shows. The wide susceptibility of the literature to these limitations motivated the development of a superior formulation of the daily and weekly labor scheduling problems. We present the formulation in the next section.

Insert Table 1 Here

---

3. A New Formulation of the daily and weekly Labor Scheduling Problems (NFLSP)

In developing NFLSP our goal was to avoid Limitations 1 and 2. As discussed below, replacing DFLSP and KFLSP's desired staffing levels with minimum reasonable staffing levels avoids Limitation 1. Also, using enough binary variables in each period to represent the nonlinear value of altering the number of staff on-hand circumvents Limitation 2.

In all the tour scheduling models discussed in this paper, including NFLSP, the only relevant labor is that being scheduled. Hence, NFLSP includes all relevant labor costs in the $-\sum_{t\in T} c_t x_t$ term, which appears in its objective function (12) below. NFLSP's objective function also includes a term, $\sum_{p\in P}\sum_{j=1} d_{pj}\mu_{pj}$, account for the incremental benefits of exceeding the "minimum reasonable staffing levels". These incremental benefits are measured as the expected present value of benefits - both present and future—that will arise only because of the current week's schedule. Those revenues and non-labor costs that are associated with simply meeting, but not exceeding, the minimum staffing levels can be viewed as constants—that is, sums that remain unchanged over the set of feasible solutions—and as such need not be represented in NFLSP's objective function. Given this preamble, NFLSP is

$$\max Z = \sum_{p\in P}\sum_{j=1}^{q_r} d_{pj}\mu_{pj} - \sum_{t\epsilon T} c_t x_t$$

subject to

$$\sum_{t\epsilon T} a_{tp}x_t - \sum_{j=1}^{q_r}\mu_{pj} \le m_p \qquad \text{for } p\epsilon P$$

$$\mu_{pj} = 0,1 \qquad \text{for } \epsilon P \text{ and } j = 1, \dots, q_p,$$

$x_t \geq 0$ and integer for $t \epsilon T$

Here m_p is the minimum reasonable number of employees to have working in period p; d_{pj} is the incremental improvement in NPV profit (ignoring labor costs) that occurs with the addition of the $(m_p + j)$th employee in period p (we assume non-increasing marginal NPV returns for each period—that is $d_{pj} \geq d_{p,j+1}$ for $p \epsilon P$ and $\forall j$); q_p is the number of employees in period p, in excess of the minimum reasonable staff size, who, ignoring labor costs, contribute to increased NPV profits (that is, $d_{pq_p} > 0$); and $\mu_{pj} = 1$, if the number of employees working in period p equals or exceeds $m_p + j$, $\mu_{pj} = 0$, otherwise.

To expand on our preamble, NFLSP's objective (12) differs from the objectives of DFLSP and KFLSP in two important aspects. First, by maximizing NPV profits, NFLSP explicitly recognizes the long-run increase (decrease) in profit that results from good (poor) customer service. Second, NFLSP's objective uses separate terms for the cost of tours and the effect of different staffing levels on customer- service-related profits. This enables NFLSP to explicitly represent the tradeoff between labor costs and the costs of poor and the benefits of good customer service. Note that the NPV of future labor costs does not appear in (12). This exclusion is correct since the NPV calculations include only the estimates of the profits and costs that one can attribute to the labor scheduling decisions for the current week—not those associated with labor scheduling decisions made in future weeks.

Constraint set (13) ensures that the minimum reasonable number of staff is present in each period. Binary overstaffing variables (μ_{pj}) measure increases from the minimum reasonable staffing level. If increasing the number of staff in a period by one person improves NPV profits, then an additional binary variable is appropriate for the period. Note that the *staff-addition* *coefficients*—the coefficients representing the value of adding staff to a period (the d_{pj})—do not include the labor cost of increasing the staff size in a period. The cost of labor arises from the scheduled tours, and so one cannot determine it before developing the labor schedule. A formulation can only avoid Limitation 1 by keeping the cost of labor separate from cost of poor and the benefit of good customer service.

NFLSP applies in any environment in which one can estimate the present-value benefits of different staff sizes. This estimation is likely to be easier if one can use queuing models of the system when making the evaluation. However, one can also use simulation to evaluate the expected benefits of alternate staff sizes in non-standard systems. This means, for example, that NFLSP is not limited to use only in systems with a one-to-one matching of service personnel to customers.

An important element of NFLSP is each period's minimum reasonable staff size. For three reasons we initially define a period's minimum reasonable staff size as the smallest number of employees who have, in aggregate, a customer service rate that exceeds the expected arrival rate of customers in the period. First, if fewer employees are present in the period, the organization will clearly provide very poor customer service. Second, we presume that managers will commonly derive NFLSP's staff-addition coefficients using information originating from a queuing model of the service delivery system. Doing this is more straightforward when the system is stable—when at least the minimum reasonable number of staff is present. Finally, since there is little evidence in the literature to indicate otherwise, we presume that managers and researchers determine the desired staff sizes independently across periods. The resultant implicit assumption of interperiod independence has greater validity in a stable system (when at least the minimum reasonable number of staff is present in a period). Section 6 addresses the effect of defining the minimum reasonable staff size differently.

Consider a situation where adding an employee in a period increases customer-service-related NPV profits by more than the cost of the least expensive tour covering the period. In this event, one can reduce the number of binary overstaffing variables in NFLSP, without altering the true nature of the model, simply by increasing the minimum reasonable staff size for the period in question. To state this formally, if

$$d_{pj} \geq \min_{(t \epsilon T | a_{tp} = !)} c_t$$

then increase m_p by j employees and appropriately reduce the number of binary overstaffing variables NFLSP contains. This reduction in the number of binary overstaffing variables is useful tor limiting NFLSP’s size growth as more tour variables, representing increased flexibility (and less costly tours), are added.

Appendix A presents an example of NFLSP for a simple shift-scheduling problem. The example illustrates how one can apply Eq. (16) to reduce the number of binary overstaffing variables in NFLSP. For comparative purposes, Appendix A also presents examples of DFLSP and KFLSP.

Insert Table 2 Here

---

4. Design of the model validation experiment

To evaluate NFLSP’s effectiveness compared to DFLSP and KFLSP, we devised a comprehensive experiment simulating service delivery systems. This section identifies assumptions inherent in the simulation experiment, provides an overview of the experimental design, describes the customer arrival- and customer service-related experimental factors, examines the issue of information accuracy, discusses formulation-related issues, and furnishes details on the simulation.

4.1. Simulation assumptions

To develop test environments in which to validate the new model, one must, by necessity, make certain assumptions and choices. In making these decisions, we used the guides of practically and the goal of developing a diverse set of test environments.

We chose a standard M / M / c / ∞ queuing system as the basic test environment. Assumptions of such systems include (1) customer service times and customer interarrival times follow exponential distributions; (2) no customers balk from the service system; and (3) customers wait in a single, first-in-first-out queue. We also assumed that (4) the operating horizon was a seven-day week, where each day was comprised of 18 hour-long planning periods; (5) the long-term mean arrival rate was 0.75 customers per minute; (6) customers were very satisfied, satisfied, or dissatisfied with the service, based on the time they spent waiting for service to begin; (7) the organization lost future revenue for each customer dissatisfied with the service and gained future revenue for each customer very satisfied with the service; (8) employees worked as scheduled; (9) all tours were comprised of five consecutive work days, with each daily shift starting at the same time and consisting of a four-hour work stretch, an hour-long break, and another four-hour work stretch; and (10) all seventy unique tours were of equal cost, per working (nonbreak) period.

4.2. Overview of the experimental design

The experiment had a full-factorial design, with ten factors, as identified in Table 2. The factors were the true daily customer arrival-rate pattern; the within-day variation in the true daily customer arrival-rate pattern; the within-week variation in the mean daily arrival rate; the extent of unpredictable variability in mean daily arrival rate; the mean duration of customer service; the true functional relationship between a customer's waiting time and their probability of being very satisfied, satisfied, or dissatisfied with the service; the NPV cost (loss of profit) incurred from dissatisfying a customer; the extra NPV profit incurred from making a customer very satisfied with the service; the accuracy of the estimates of the cost of customer dissatisfaction and of the profit from very satisfied customers; and the labor scheduling problem formulation. We included the first nine factors in the experiment, which are environmental factors, because we felt that they could potentially influence the relative performance of the three labor scheduling formulations (the tenth factor). Moreover, the variability in the simulated service delivery systems resulting from the diversity of the environmental factors ensures that the best performing problem formulation will be broadly effective in service organizations, and not just effective in a restricted environment that may be peculiar in some regard.

4.3. Customer arrival-related, environmental, experimental factors

There were three within-day customer arrival-rate patterns: unimodal, with one daily arrival peak; bimodal, with two daily arrival peaks; and trimodal, with three daily arrival peaks. Respectively, such patterns occur in facilities with mid-day peak demand, such as water parks, facilities like dry cleaners, with commuter-driven demand, and facilities with demand related to meal-times, like restaurants. We used sinusoidal functions to model the instantaneous arrival rate, since the within-day arrival-rate variation is easily controlled using the functions' amplitude. The second customer arrival rate factor was the within-day variation in the customer arrival rate. Its two levels were coefficients of variation in the true daily customer arrival-rate pattern of 0.25 and 0.50 (sine function amplitudes of 0.353 and 0.706, respectively). The third customer arrival-related experimental factor is the within-week variation in the mean daily arrival rate. Level one of this factor had equal mean arrival rates on all days, while level two had relative mean arrival rates of 1.0, 0.7, 0.7, 1.0, 1.0, 1.3 and 1.3 for Sunday through Saturday, respectively. The final customer arrival-related experimental factor is the extent of unpredictable variability in the mean daily arrival rate. Its two levels have coefficients of variation of 0.05 and 0.20 in mean daily arrival rates.

Fig. 1 illustrates examples of the simulated number of customer arrivals for a single week, by hour, for the Unimodal arrival-rate pattern with the low levels of within-day and within-week arrival rate variation, under the two levels of unpredictable variation in the mean daily arrival rate. The customer arrival-rate patterns exhibit a consistent shape (the factors representing the arrival-rate pattern, within-day arrival-rate variation, and within-week arrival-rate variation) because of what determines when customers place demand on the service. Variability in mean daily arrival rates (the factor representing the unpredictable variation in the mean daily arrival rate) occur due to unpredictable causes. Consider, for example, traffic arriving at a commuter-highway toll booth. If one plots the number of car arrivals per 15 minute interval over the 96 daily intervals, for four successive Wednesdays, one will most likely see a consistent pattern emerging (represented by the first three arrival rate-related factors). Despite this consistent shape, the total number of arrivals per day is likely to vary over the Wednesdays (represented by the last arrival rate-related factor). This may be a result of accidents on alternate highways, more commuters using public transit, or fewer commuters carpooling—all of which vary from day to day and consequently would be difficult to predict.

Insert Figure 1 Here

4.4., Customer service-related, environmental, experimental factors

The first customer service-related experimental factor, the mean service duration, had two levels: 1 and 10 minutes per customer. The second customer service-related factor represented the true functional relationship between a customer's waiting time and their probability of being very satisfied, satisfied, or dissatisfied with the service. This factor had two levels. Fig. 2 illustrates what we choose to call the *gradual* and the *rapid* waiting-time/satisfaction functional relationships. We use logistic functions to describe these relationships, as it appears Davis (1991) did (see his Fig. 3, p. 427). The probability of a customer being dissatisfied with the service given a wait of w minutes, $\mathrm{PD}(w)$, and the probability that a customer is very satisfied with the service given a wait of w minutes, $\mathrm{PVS}(w)$, are

$$\mathrm{PD}(w) = (1 + be^{-cw})^{-1}$$

$$\mathrm{PVS}(w) = 1 - (1 + de^{-cw})^{-1}$$

where

$$b = \begin{cases} 403.429, \text{ with the gradual waiting-time/satisfaction relationship} \\ 3{,}269{,}020, \text{ with the rapid waiting-time/satisfaction relationship} \end{cases}$$

$$c = \begin{cases} 4, \text{ with the gradual waiting-time/satisfaction relationship} \\ 10, \text{ with the rapid waiting-time/satisfaction relationship} \end{cases}$$

$$d = \begin{cases} 7.38905, \text{ with the gradual waiting-time/satisfaction relationship} \\ 148.413, \text{ with the rapid waiting-time/satisfaction relationship} \end{cases}$$

We selected the particular parameter values in Eqs. (17) and (18) to ensure that the functional relationships are similar in two regards: both have 50% of customers very satisfied with a 0.5 minute wait for service, and both have 50% of the customers dissatisfied with a 1.5 minute wait for service.

Insert Figure 2 Here

The third customer service-related experimental factor was the relative cost to the organization of a dissatisfied customer. This factor had two levels: 1 and 5 NPV labor-hour-equivalents, or LHEs, where a labor-hour-equivalent is the average labor cost, per hour worked. The fourth customer service-related experimental factor represents the NPV profit to the organization of making a customer very satisfied with the service. Its two levels had NPV profits of 1 and 5 LHEs. The final customer service-related experimental factor was the accuracy of the estimates of the dissatisfaction cost and of the extra-satisfaction profit. Its three levels represented inaccuracy in the estimates of - 30%, 0% and + 30%. For example, with the low levels of dissatisfaction cost, extra-satisfaction profit, and cost/profit estimation accuracy, while the true cost of a dissatisfied customer is one LHE and the true additional profit from a very satisfied customer is also one LITE, these values would each be estimated to be 0.7 LHEs. The effect of underestimating the dissatisfaction cost and extra-satisfaction profit is to specify fewer than the desired number of staff, while overestimating the values has the opposite effect.

4.5. Information accuracy

Since, in practice, staffing levels used as inputs to the labor scheduling process originate with forecasts of customer arrivals and service durations, the accuracy of these forecasts is a concern. Our assumptions of a stable long-term mean customer arrival rate and of a stable mean customer service rate allowed us to mimic inaccuracy in the forecasts of customer arrival and service durations—without the complication of actually forecasting—by using a limited amount of simulated historical information. We obtained the expected customer arrival and service rates using data from twenty weeks of simulated historical operation of the service delivery function. In practice, it is also highly unlikely that one would know the true waiting-time/satisfaction functional relationship with certainty. By surveying their facility's customers, managers may develop a histogram relating the probability of dissatisfaction to the time customers spend waiting for service (see (Davis, 1991)). We assumed the sampling of five hundred customers, with equal numbers of customers falling in twenty waiting-time intervals.[2] Thus, there would be 25 observations, or customers sampled, from each category. Figs. 2a and 2b show the true, but for practical purposes unknown, proportion of customers who are very satisfied, satisfied, and dissatisfied with the service. Figs. 2c and 2d also show an example of the proportion of customers falling in these categories based on the assumed sampling of 500 customers.

4.6. Formulation-related issues

[2] These intervals were 0,000-0.125, 0.125-0.150,..., 2.375-2.500 for the gradual waiting-time/satisfaction functional relationship and 0.000-0.100, 0.100-0.200,…, 1.900-2.000 for the rapid waiting-time/satisfaction functional relationship.

In this section we address issues related to developing schedules using each problem formulation. Specifically, we address the setting of employee requirements in DFLSP and KFLSP, NFLSP's staff-addition coefficients, and the models' heuristic solution procedure.

4.6.1. Setting the employee requirements in DFLSP and KFLSP

We followed an iterative, eight-step process to identify each period's employee requirement:

1. Adjust each period's expected arrival rate, using the procedure of Thompson (1993), to account for the spillover of service across periods.
2. Identify the minimum reasonable number of employees for the period.
3. With the gradual waiting-time/satisfaction functional relationship, use an appropriate queuing model ($M / M / c / \infty$ for our experiment) to find the probability of a customer's waiting time falling in each interval 0.000-0.125, 0.125-0.150,…, 2.250-2.375, 2.375-∞. With the rapid waiting- time/satisfaction functional relationship, use an $M / M / c / \infty$ queuing model to find the probability of a customer's waiting time falling in each interval 0.000-0.100, 0.100-0.200....... 1.800-1.900, 1.900-∞
4. For each interval identified in step three, multiply the probability that a customer's waiting time falls in the interval by the probability of customer's dissatisfaction with a wait for service falling in the interval (from the customer dissatisfaction function histogram).
5. Sum the quantity found in step four across all intervals, yielding an expected probability of dissatisfaction, and then multiply the total by the expected number of customer arrivals

in the period and by the estimated dissatisfaction cost. This yields an expected total dissatisfaction cost for the period with the given staff size.

6. Repeat steps four and five, this time finding, for the given staff size, the expected total benefit of making customers very satisfied with the service.
7. Subtract the expected total dissatisfaction cost and the approximate cost of labor for the period from the expected total benefit accruing from very satisfied customers. (Since one only knows the true average cost of tours scheduled to cover the period *after* developing the labor schedule, we assume that each employee added in the period costs a single LHE as per our discussion in Section 2.2.)
8. Repeat steps 3-7 for larger staff sizes to find the greatest total NPV profit for the period. The staff size yielding the greatest total NPV profit becomes the period’s employee requirement (Appendix A presents an example of steps 6 and 7.)

4.6.2. NFLSP's staff-addition coefficients

In determining the staff-addition coefficients in NFLSP, we first followed, for each planning period, steps 1-5 from the procedure used in setting DFLSP and KFLSP’s employee requirements. Subtracting the expected total dissatisfaction cost from the expected total benefit accruing from very satisfied customers gave the expected, customer-service-related NPV profit for a period with a given staff size. The staff-addition coefficients for each period came from calculating the net change in customer-service-related NPV profit that occurred with incremental increases in the number of staff working in a period.

4.6.3. Heuristic solution procedure

For several reasons, we used a heuristic solution methodology to generate schedules in the problem environments. First, practitioners and researchers have commonly used heuristics to generate tour schedules, largely due to the difficulty in solving such problems optimally. Second, the sheer number of problem environments we consider makes it impractical to generate optimal schedules (we would have to generate optimal schedules to 3,456 tour problems).

We developed solutions to the models using a simulated annealing heuristic based on that developed by Brusco and Jacobs (1993). In an extensive study of tour scheduling heuristics, Bechtold et al. (1991) found the heuristics of Keith (1979) and Morris and Showalter (1983) to perform best. Brusco and Jacobs (1993) simulated-annealing heuristic outperformed both of these heuristics, however. Appendix B provides details about our implementation of the simulated-annealing heuristic.

4.7. Simulation details

The process observed in conducting the simulation is as follows:

1. Select some combination of arrival-rate pattern, within-day arrival-rate variation, within-week arrival-rate variation, and unpredictable arrival-rate variation. Generate 20 historical and 50 future weeks of information on customer arrivals. Also, randomly sample 500 customers for each waiting-time/satisfaction functional relationship.
2. Select some combination of mean service duration, waiting-time/satisfaction functional relationship, dissatisfaction cost, extra-satisfaction profit, cost/profit: estimation accuracy, and problem formulation.
3. Develop the employee requirements (or minimum reasonable staff sizes and the staff-addition coefficients) based on the twenty weeks of historical information.

4. Heuristically generate a tour schedule.
5. Simulate the operation of the facility for 50 weeks with the tour schedule developed in step four in effect and collect relevant information on the average total weekly NPV profit resulting from implementing the schedule.
6. Repeat steps 2-5 for all combinations of the levels of mean service duration, waiting-time/satisfaction functional relationship, dissatisfaction cost, extra-satisfaction profit, cost/profit estimation accuracy, and problem formulation.
7. Repeat steps 1-6 for all combinations of the levels of arrival-rate pattern, within-day arrival- rate variation, within-week arrival-rate variation, and unpredictable arrival-rate variation.

To summarize, the number of experimental factor levels resulted in a total of 1,152 service delivery environments. For each environment, we developed and evaluated a schedule for each problem formulation, resulting in the experiment having 3,456 observations. With the simulation model coded in FORTRAN, running the experiment took approximately 90 hours on a Pentium 90-based personal computer.

Readers should not construe our use of 50 simulated weeks of operation with the same schedule in effect as our advocating keeping the same schedule in effect for a period longer than a single week. Simulating 50 weeks of operation serves only to identify the expected (average) profit associated with implementing the schedule for a single week.

5. Results of the model validation experiment

Table 3 summarizes results of the experiment. On average, the weekly schedules generated using NFLSP were 82.16 LHEs and 341.52 LUEs more profitable than those of

KFLSP and DFLSP, respectively, differences significant at the 0.001 level. If one assumes that the average hourly wage rate is $6.00, including benefits, and that a service facility operates 52 weeks a year, then NFLSP's average advantage over KFLSP would translate to an additional yearly profit of $24,648. NFLSP would provide $96,046 more than DFLSP under the same conditions. NFLSP's schedules were 2.2% and 0.6% more profitable than those of DFLSP and KFLSP, respectively. The mean labor costs of DFLSP's, KFLSP's and NFLSP's schedules were 2095.79, 1549.88 and 1663.50 LHEs, respectively, as Table 3 reports.

To evaluate how the heuristic solution procedure affects the NPV profits, we attempted to solve a subset of the tour problems optimally using commercially-available IP (integer programming) software. We randomly selected 20 test environments from the complete set of 1,152 test environments and attempted to solve the integer programming version of the three formulations. We generated the models using GAMS (IBM Corporation, 1991) and solved the models with OSL (Brooke et al., 1992). We imposed a 15-minute time limit for the model solution, and recorded the best integer solution found during that period. Table 3 reports how these solutions differ from the heuristic solutions for the same 20 test environments.

Insert Figure 3 Here

Insert Table 3 Here

We developed an ANOVA model with average total weekly NPV profit as the dependent variable. This model included all main factor effects and all first- and second-order interaction terms. With two exceptions, all problem-formulation-related first-order interaction terms were significant at the 0.001 level. The exceptions, which were not significant at the 0.10 level, were the interaction of the problem formulation with the within-week variation in the mean daily arrival rate and with the waiting-time/satisfaction functional relationship. Fig. 3 illustrates the average and relative NPV profits, per week, by levels of problem formulation and the environmental factors. This figure shows that for all levels of all experimental factors, NFLSP's schedules had greater average profits than those of KFLSP, which in turn had greater average profits than those of DFLSP. NFLSP's profit advantage relative to KFLSP and DFLSP was greater under the bimodal arrival-rate pattern, the lower level of within-day arrival-rate variation, and the longer mean service duration. NFLSP's profit advantage was greater compared to DFLSP but lower compared to KFLSP under the higher level of unpredictable variation in the mean daily arrival rate and when the dissatisfaction cost and extra-satisfaction profit were overestimated.

We attempted to identify the specific conditions under which DFLSP and KFLSP performed best and worst compared to NFLSP. Since there was no significant interaction between the problem formulation and the within-week arrival-rate variation or the waiting-time/satisfaction functional relationship, we averaged the models' profits tor the tour combinations of levels of these environmental factors. Table 4 reports the findings of this analysis. At their best, NFLSP's schedules were 767.01 and 1349.26 LHEs more profitable than those of DFLSP and KFLSP, respectively. Assuming a $6.00 per hour labor cost and year-round operation, this translates into NFLSP being $239,300 and $421,000 more profitable, on average, than DFLSP and KFLSP. At their worst, NFLSP's schedules were 54.80 LHEs more profitable than those of DFLSP and 107.89 LHEs less profitable than KFLSP. Overall, NFLSP generated a more profitable schedule than KFLSP in 846 (73.4%) of the test environments, while compared to DFLSP, NFLSP generated a more profitable schedule in 1,152 (100%) of the test environments.

Insert Table 4 Here

6. Discussion

In this section we present a discussion focusing on the effect of the heuristic solution procedure on the relative performance of the models and issues relating to NFLSP. The section closes with some conclusions.

6.1. The effect of the heuristic solution procedure on relative model performance

One may question whether NFLSP's profit advantage over the 1,152 test environments is real, or an artifact of the heuristic solution process. Three reasons support the former conclusion. First, we based the heuristic on the best of the existing tour-scheduling heuristics, Brusco and Jacobs (1993) simulated annealing heuristic. Second, the sheer number of test environments strengthened our confidence in NFLSP’s superiority. There were 1,152 test problem environments varying on a wide variety of characteristics. The diversity of these problems helps ensure that a model that performs well only in certain problems will not outperform a broadly effective model - in other words, it raises the likelihood that the differences in performance are real.

Finally, the results from attempting to solve a subset of the problems optimally suggest that NFLSP’s profit advantage was due to its inherent superiority. On average, the heuristically-generated solutions equaled the quality of the best solutions identified with the IP software for

DFLSP. For KFLSP and NFLSP, the heuristically-generated solutions actually were superior to the best solutions identified with the IP software (see Table 3). These results offer strong evidence that the simulated-annealing heuristic is generating solutions very close to optimal. Thus, any differences in profitability are most likely arising from inherent differences in effectiveness of the problem formulations.

6.2. Issues relating to NFLSP

In this section we address (1) why NFLSP produces superior schedules; (2) why NFLSP's superiority may be less in practice than we have observed; (3) the effect scheduling flexibility has on NFLSP's superiority; (4) considerations when determining NFLSP's staff-addition coefficients; and (5) linking NFLSP to the labor staffing problem.

6.2.1. Why NFLSP produces superior schedules

NFLSP's advantage over DFLSP and KFLSP comes because it determines the number of employees needed in each period while developing the schedule. By doing this, NFLSP can accurately weigh the tradeoff between improved customer service and increased labor costs. Consider the effect of increasing the mean service duration - doing so raises the cost of labor, but does not raise revenue nor the potential benefit of providing good customer service. One expects, then, that NFLSP would perform relatively better compared to DFLSP and KFLSP as the service duration grows, ceteris paribus, since it becomes progressively more useful to calculate the benefit of a tour, over its whole length, as NFLSP does. Figs. 3c and 3d illustrate just such a result.

NFLSP produced superior schedules despite inaccurate information. Our experiment contained uncertainty that originated in four ways. First, given the limited amount of historical information, the true long-run average customer arrival rates are not known with certainty. Second, the factor representing the unpredictable variation in the mean daily arrival-rate ensures that the daily number of customer arrivals will vary unpredictably from the true long-run average. Third, since we used a sample of customers to identify the waiting-time/satisfaction functional relationship, this relationship is uncertain. Fourth, we investigated the effect of over- and underestimating the dissatisfaction cost and extra-satisfaction profit. That NFLSP performed best despite the four types of uncertainty suggests that NFLSP's superiority did not arise from inherently superior information about the scheduling environment, but arose instead from an inherently superior way of using whatever information is available.

6.2.2. Why NFLSP's profit advantage may be less in practice than we have observed

In practice, the difference between the profitability of DFLSP's and NFLSP's schedules might be less than that observed here. The reason is that having obtained a solution to DFLSP, an astute manager will look at the staffing levels the solution provides. We have observed that when managers see substantial overstaffing in some periods, they often relax the employee requirements in some or all of those periods having no overstaffing, and then redevelop the schedule. By following this iterative process, these managers eventually obtain a schedule that better satisfies their notion of acceptability. In essence, these managers are applying a model that more resembles NFLSP than DFLSP. The clear benefit of NFLSP is that since it incorporates appropriate information on the merits of different staff sizes, one can avoid the tedious, iterative, trial-and-error approach. Moreover, the best schedule that one can hope for using the trial-and-error approach with DFLSP will not be better than the schedule generated using NFLSP.

6.2.3. The effect of scheduling flexibility on NFLSP's superiority

Since DFLSP and KFLSP both use the same employee requirements, one may wonder why they have different schedules. Indeed, their schedules would be identical if one could exactly satisfy all their employee requirements, which could happen if labor scheduling flexibility was arbitrarily high. It is when one cannot exactly satisfy all the employee requirements that NFLSP works particularly well, since NFLSP facilitates an accurate tradeoff between improved customer service and increased labor costs. As such, one may consider NFLSP itself as offering a form of labor scheduling flexibility (other forms of flexibility useful in reducing labor scheduling costs are more shift length alternatives (Bailey and Field, 1985; Henderson and Berry, 1976, 1977; Jacobs and Bechtold, 1993; Mabert and Watts, 1982; Showalter and Mabert, 1988), more shift starting times (Henderson and Berry, 1976, 1977; Mabert and Watts, 1982), greater variability in shift start times across days in weekly schedules (Bailey, 1985; Jacobs and Bechtold, 1993), more choice in the number of days worked per week (Jacobs and Bechtold, 1993; Showalter and Mabert, 1988), and more times at which t j take breaks (Bechtold and Jacobs, 1990, 1991; Jacobs and Bechtold, 1993). Given this, we hypothesize that NFLSP will increase its relative superiority when scheduling a pool of employees who are available for work only during specific periods (see, for example, (Thompson, 1990)), since the limited availability of employees reduces scheduling flexibility.

6.2.4. Considerations when determining the NFLSP's staff-addition coefficients

Because our simulation experiment allowed no customer balking, it is reasonable, for the reasons addressed in Section 3, to define the minimum reasonable staff size initially as the smallest number of employees who, in aggregate, can serve the expected number of customers.[3] Clearly, there are situations where fewer employees would be appropriate and perhaps increase schedule profitability. For example, if planning intervals are very short, the effect of a period having an arrival rate that exceeds the aggregate service rate is not likely to be dramatic, particularly if the aggregate service rate exceeds the arrival rate in subsequent periods. *NFLSP only requires staff-addition coefficients be calculated in a manner appropriate for the type of queuing system existing in the service facility, whatever minimum reasonable staffing level one chooses to use.*

In our simulation, we ignored the direct profit of serving the customers arriving each day.[4] Since our experiment assumed that every customer who arrived would be served, the direct revenue from serving customers was irrelevant in the staffing decision. There are situations where it would not be desirable to ignore the direct profit of serving customers arriving each day; for example, when customer balking can occur. Although NFLSP needs no modification for these situations, one would have to measure *all* relevant NPV profits associated with customer service when calculating the staff-addition coefficients.

Caution is necessary when calculating the staff-addition coefficients, because NFLSP assumes monotonically decreasing staff-addition coefficients for each planning period, or that $d_{p1} \geq d_{p2} \geq d_{p3} \geq \cdots$

[3] Recall that the minimum staff size may be subsequently increased using Eq. (16).
[4] Recall that we only accounted for the increased future profit from very satisfied customers and for the lost future profit from dissatisfied customers.

Given the nonlinear improvement in queuing system performance that occurs when adding staff, Eq. (19) must apply, *if one calculates the staff-addition coefficients correctly*. For example, if one uses *mean* customer waiting time with the waiting-time/satisfaction functional relationships, one may perhaps measure little change in NPV profitability when adding the first few employees (above the minimum reasonable staff size). With our waiting-time/satisfaction functional relationships defined by Eqs. (17) and (18), NPV profits will increase only if the mean waiting time drops below approximately 2.5 minutes. It can be important, therefore, to use multiple waiting-time intervals, as we have done, and, using an appropriate queuing or simulation model, determine the expected number of customers falling in each waiting time interval. In turn, one can use the distribution of customers across waiting-time intervals with information about customer satisfaction to determine the true effect on the organization's profit (that is, to calculate the staff-addition coefficients accurately).

6.2.5. Linking NFLSP to the labor staffing problem

This paper has focused upon labor scheduling. Labor scheduling assumes that a fixed pool of employees is available. A longer-term issue is labor staffing, which determines when to change the size or the mix of the employee pool. Clearly, one could link DFLSP and KFLSP to the labor staffing problem by incorporating the costs of changing the size of the work-force into the model. Given its superiority over DFLSP and KFLSP because of its accurate incorporation of the true benefits (costs) of good (poor) service, NFLSP offers managers a much better basis for linking the labor staffing and scheduling problems. Developing this linkage is a possible extension of our research.

6.3. Conclusions

This paper has presented a new formulation of the daily and weekly labor scheduling problems. The new formulation proved superior to existing formulations in an extensive service delivery system simulation experiment. Although NFLSP’s benefit to any specific facility will depend on the facility’s service characteristics, its broadly superior performance justifies its adoption by both researchers and practitioners. Research extensions suggested by our results include reevaluating the effectiveness of heuristic solution procedures using the new model, linking the new model to the labor staffing decision, and evaluating the relative performance of the model in environments having reduced labor scheduling flexibility—for example, in situations where employees are available for work only at limited times.

Appendix A

In this appendix we present an example of DFLSP, KFLSP and NFLSP for a simple shift-scheduling problem. We assume there are five planning periods, and that shifts are three periods long. We also assume that the marginal benefit of different staff sizes has been evaluated using an appropriate simulation or queuing model, resulting in the data reported in Table 5.

Table 5 shows the minimum number of employees required to serve the customers expected to arrive each period. This is two employees in period two and one employee in all other periods. We presume that the desired staff size for DFLSP and KFLSP would be set independently for each period at the point giving the highest marginal contribution, while assuming that each employee working the period costs a single LHE. This gives desired staff sizes of 2, 4, 3, 2 and 3 employees for periods one through five, respectively.

Define s_p as the number employees working a shift that starts in period p. Since each shift costs three LHEs, DFLSP is

$$\min Z = 3s_1 + 3s_2 + 3s_3$$

subject to:

$$s_1 \geq 2, \qquad s_1 + s_2 \geq 4, \qquad s_1 + s_2 + s_3 \geq 3,$$

$$s_2 + s_3 \geq 2, \qquad s_3 \geq 3$$

$$s_1, s_2, s_3 \geq 0 \; and \; integer.$$

In formulating KFLSP for this problem, one cannot reasonably apply the same costs of under- and overstaffing as described in Section 2.1. The reason is that the costs of understaffing reported in Section 2.1 are so high compared to costs of the shifts that no understaffing will occur (and thus KFLSP would yield essentially the same schedule as DFLSP). So, for this example, we set the cost of the unbounded understaffing such that two periods of this understaffing would justify an additional shift but one period of this understaffing would not (thus giving a cost of 1.55 LHEs). Also, we set the cost of the bounded understaffing so that three periods of this understaffing would justify an additional shift but two would not (thus giving a cost of 1.05 LHEs). Finally, we arbitrarily set the cost of unbounded overstaffing to 0.15 LHEs. With these costs, and applying the bounds imposed in (11), KFLSP for this problem is

$$\text{Min } Z = 3s_1 + 3s_2 + 3s_3 + 1.55\alpha_2 + 1.55\alpha_3 + 1.55\alpha_5 + \sum_{p=1}^{5} (1.05\beta_p + 0.15\sigma_p),$$

subject to

$$s_1 + \beta_1 - \sigma_1 - \pi_1 = 2,$$
$$s_1 + s_2 + \alpha_2 + \beta_2 - \sigma_2 - \pi_2 = 4,$$
$$s_1 + s_2 + s_3 + \alpha_3 + \beta_3 - \sigma_3 - \pi_3 = 3,$$
$$s_2 + s_3 + \beta_4 - \sigma_4 - \pi_4 = 2,$$
$$s_3 + \alpha_5 + \beta_5 - \sigma_5 - \pi_5 = 3,$$
$$\alpha_3, \alpha_5, \beta_1, \ldots, \beta_5, \sigma_1, \ldots, \sigma_5 \leqslant 1,$$
$$\alpha_2 \leqslant 2,$$
$$s_1, s_2, s_3 \geqslant 0 \text{ and integer.}$$

Without regard to Eq. (16), NFLSP for this problem is

$$
\begin{aligned}
\text{Max } Z = {} & 8.280\mu_{11} + 0.757\mu_{12} + 0.068\mu_{13} \\
& + 9.271\mu_{21} + 1.384\mu_{22} + 0.205\mu_{23} \\
& + 0.029\mu_{24} + 14.21\mu_{31} + 1.417\mu_{32} \\
& + 0.148\mu_{33} + 0.014\mu_{34} + 8.575\mu_{41} \\
& + 0.776\mu_{42} + 0.069\mu_{43} \\
& + 11.804\mu_{51} + 1.142\mu_{52} \\
& + 0.113\mu_{53} + 0.010\mu_{54} - 3s_1 - 3s_2 - 3s_3,
\end{aligned}
$$

subject to

$$
\begin{aligned}
& s_1 - \mu_{11} - \mu_{12} - \mu_{13} \geqslant 1, \\
& s_1 + s_2 - \mu_{21} - \mu_{22} - \mu_{23} - \mu_{24} \geqslant 2, \\
& s_1 + s_2 + s_3 - \mu_{31} - \mu_{32} - \mu_{33} - \mu_{34} \geqslant 1, \\
& s_2 + s_3 - \mu_{31} - \mu_{42} - \mu_{43} \geqslant 1, \\
& s_3 - \mu_{51} - \mu_{52} - \mu_{53} - \mu_{54} \geqslant 1, \\
& s_1, s_2, s_3 \geqslant 0 \text{ and integer}, \\
& \mu_{11}, \mu_{12}, \mu_{13}, \mu_{21}, \mu_{22}, \mu_{23}, \mu_{24}, \mu_{31}, \mu_{32}, \\
& \quad \mu_{33}, \mu_{34}, \mu_{41}, \mu_{42}, \mu_{43}, \mu_{51}, \mu_{52}, \mu_{53}, \mu_{54} \leqslant 1.
\end{aligned}
$$

Recognizing that each shift costs three LHEs, one can reduce the number of binary overstaffing variables in NFLSP by applying Eq. (16). This results in the elimination of any variables with a monetary benefit exceeding 3 LHEs and an appropriate increase in the employee requirements.

Doing this yields

$$\begin{aligned}\text{Max } Z = {} & 0.757\mu_{11} + 0.068\mu_{12} + 1.384\mu_{21} \\ & + 0.205\mu_{22} + 0.029\mu_{23} + 1.417\mu_{31} \\ & + 0.148\mu_{32} + 0.014\mu_{33} + 0.776\mu_{41} \\ & + 0.069\mu_{42} + 1.142\mu_{51} + 0.113\mu_{52} \\ & + 0.010\mu_{53} - 3s_1 - 3s_2 - 3s_3,\end{aligned}$$

subject to

$$s_1 - \mu_{11} - \mu_{12} \geqslant 2,$$
$$s_1 + s_2 - \mu_{21} - \mu_{22} - \mu_{23} \geqslant 3,$$
$$s_1 + s_2 + s_3 - \mu_{31} - \mu_{32} - \mu_{33} \geqslant 2,$$
$$s_2 + s_3 - \mu_{41} - \mu_{42} \geqslant 2,$$
$$s_3 - \mu_{51} - \mu_{52} - \mu_{53} \geqslant 2,$$
$$s_1, s_2, s_3 \geqslant 0 \text{ and integer},$$
$$\begin{aligned}& \mu_{11}, \mu_{12}, \mu_{13}, \mu_{21}, \mu_{22}, \mu_{23}, \mu_{24}, \mu_{31}, \mu_{32}, \\ & \quad \mu_{33}, \mu_{34}, \mu_{41}, \mu_{42}, \mu_{43}, \mu_{51}, \mu_{52}, \mu_{53} \leqslant 1.\end{aligned}$$

The optimal solution to DFLSP requires seven shifts (a, = 2, a2 = 2 and a, = 3). KFLSP's optimal solution requires four shifts ($s_1 = 2$, $s_2 = 2$ and $s_3 = 3$). Finally, NFLSP's optimal solution requires five shifts ($s_1 = 2$, $s_2 = 0$ and $s_3 = 2$). Assuming the marginal benefits of increasing the staff size reported in Table 5 are accurate, the true profitability of DFLSP's, KFLSP's and NFLSP's schedules are 36.092, 32.430 and 39.493 LHEs, respectively. This example shows that DFLSP may schedule too many staff, while KFLSP's may schedule too few staff (Table 3 reports similar results from the large experiment). NFLSP alone enables an accurate evaluation of the merits of additional shifts or tours.

Appendix B

This appendix describes the simulated annealing heuristic used in generating solutions to DFLSP, KFLSP and NFLSP. The heuristic derives from the one developed by Drusco and Jacobs (1993). Define X as the current base schedule, X^b as the best schedule found, and X' as the current perturbed solution. Also, define Z as the objective value of the current base schedule, Z^b as the objective of the best schedule found, Z' as the objective value of the current perturbed solution, ctime as the current time and xtime as the maximum time (=30 seconds). Our implementation of Brusco and Jacobs (1993) heuristic is

1. Add employees to tours to obtain a feasible schedule and then eliminate redundant employees to get an initial schedule X.
2. Set $X^b = X$. Set $Z^b = Z$. Set $j = 0$, $k = 60$ and $r = 0.95$. If the model is DFLSP, set $t = 50$; otherwise set $t = 1.25$.
3. Set n is equal the number of employees scheduled in X and $j = j + 1$. If $j \leqslant k$, proceed to step 4; otherwise proceed with step 8.
4. If $j < k$, set $f = \min(15, n/3)$; otherwise, set $f = \min(n, \max(15, n/5))$. Perturb X by dropping f employees, adding employees to tours to obtain a feasible schedule, and eliminating redundant employees to obtain schedule X'.
5. If the model is DFLSP or KFLSP and $Z' < Z^b$ or the model is NFLSP and $Z' > Z^b$, then set $X^b = X'$, $X = X'$, $Z^b = Z'$ and $Z = Z'$ and go to step 7; otherwise continue.
6. If the model is DFLSP or KFLSP set $\delta = Z - Z'$; otherwise set $\delta = Z' - Z$. Set y equal to a random [0,1) variate. If $\delta \geqslant 0$ or $y \leqslant e^{\delta/t}$ then set $X = X'$ and $Z = Z'$.
7. If ctime > xtime STOP; otherwise return to step 3.
8. Set $j = 0$ and $t = t * r$, and return to step 3.

When adding employees, the heuristic selected to add an employee to the tour having the highest value of

$$\begin{bmatrix}\text{improvement in the profit or cost associated}\\ \text{with overstaffing and understaffing}\end{bmatrix} - [\text{cost of the tour}], \qquad \textbf{(B.1)}$$

with any ties broken randomly. For all models, there was an arbitrarily high benefit (M) associated with adding an employee to a period with an unsatisfied employee requirement restriction.[5] Once a model's employee requirement restrictions were satisfied in all periods, the heuristic continued to add employees to tours providing Eq. (B.1) exceeded zero.

When eliminating redundant employees, each employee is examined to see if the tour he/she is scheduled to work can be dropped with a positive monetary benefit and without violating the model's employee requirement restrictions. Some redundant employee is selected at random and dropped from the schedule, and then the schedule is again checked for redundant employees. The process of checking for and randomly dropping a redundant employee repeats until the schedule contains no redundant employees.

The process of dropping f employees from the schedule in undertaken in three phases, each phase dropping $f/3$ employees. Each phase starts with an evaluation of the net benefit of dropping the remaining employees. The heuristic identifies the $2f/3$ employees who, if dropped, least increase the cost of the schedule. From this group, $f/3$ employees are randomly selected to be eliminated. The rationale for the three-phase process is that it avoids having to re-evaluate the cost implications of dropping every employee each time an employee is dropped, thus saving a substantial amount of computational effort. As with adding employees to tours, we impose an

[5] For DFLSP, this means having fewer than r_p employees in period p; for KFLSP, it means having fewer than one employee in a period; and for NFLSP, it mean having fewer than m_p employees in period p.

arbitrarily high cost associated with each employee- period violation of a model’s employee requirement restrictions.

Table 1. A classification of germane labor scheduling literature

Labor scheduling formulation [a]	References	Applicable shortcomings [b]	
		Limitation 1	Limitation 2
DFLSP	(Bailey and Field, 1985; Baker et al., 1973; Bartholdi, 1981; Bechtold and Jacobs, 1990, 1991; Bechtold and Showalter, 1985, 1987; Bechtold et al., 1991; Brusco and Jacobs, 1993; Dantzig, 1954; Easton and Rossin, 1991a, 1991b; Gaballa and Pearce, 1979; Henderson and Berry, 1976, 1977; Jacobs and Bechtold, 1993; Li et al., 1991; Loucks and Jacobs, 1991; Mabert and Showalter, 1990; Moondra, 1976; Morris and Showalter, 1983; Segal, 1974; Showalter and Mabert, 1988; Thompson, 1992)	Yes	Yes
		Yes	Yes
KFLSP	(Keith, 1979; Krajewski et al., 1980 [c]; Mabert, 1979; Mabert and Watts, 1982 [c])	Yes	Yes
KFLSP-NBV	(Bailey, 1985; Buffa et al., 1976; McGinnis et al., 1978 [d]; Taylor and Huxley, 1989; Thompson, 1990)	Yes	Yes
KFLSP-NUV	(Baker, 1976)	Yes	Yes

Notes:

[a] The formulation closest in form to that used in the reference. DFLSP is the formulation of Dantzig (1954). KFLSP is the formulation of Keith (1979). NBV is a model allowing unbounded employee shortages and surpluses but lacking variables representing bounded employee shortages and surpluses. NUV is a model allowing bounded employee shortages and surpluses but lacking variables representing unbounded employee shortages and surpluses.

[b] Limitation 1 is the difficulty of setting the desired staffing sizes in each period so as to maximize profits. Limitation 2 is assuming that a surplus employee is of equal value across all periods.

[c] These researchers examined the scheduling of personnel to process checks in a bank. For each period, they only allowed overstaffing to process a pre-existing inventory of unprocessed checks.

[d] McGinnis et al. (1978) apparently allowed understaffing, but measured performance only using idle time (overstaffing).

Table 2. Independent variables in the service delivery system simulation experiment.

Factor	Measured as	Number of levels: levels
Within-day customer arrival-rate pattern	A sinusoidal pattern in the true underlying costumer arrival rate	3: Unimodal (one daily peak), Bimodal (two daily peaks), Trimodal (three daily peaks)
Variation in the daily customer arrival-rate pattern	A coefficient of variation in the customer arrival-rate pattern	2: 0.25 and 0.50 (sine function amplitudes of 0.353 and 0.706)
Variation in the weekly customer arrival-rate pattern	The relative mean arrival rates for the days in the week	2: equal mean arrival rates on all days, and relative mean arrival rates of 1.0, 0.7, 0.7, 1.0, 1.0, 1.3 and 1.3 for Sunday through Saturday, respectively
Unpredictable variability	The coefficient of unpredictable variation in the daily arrival rate	2: 0.05, 0.2
Mean service duration	Minutes per customer	2: 1 and 10
True functional relationship between customer satisfaction and waiting times	A curve relating the probability of a customer's being very satisfied, satisfied, or dissatisfied to their waiting time	2: Gradual and rapid (see Fig. 2)
Relative NPV cost (lost profit) of dissatisfying a customer	Number of labor-hour equivalents	2: 1 and 5
Relative NPV profit from a very satisfied customer	Number of labor-hour equivalents	2: 1 and 5
Cost/profit estimate accuracy	Proportion by which the cost of a dissatisfied customer and profit from a very satisfied customer is over or underestimated	3: −0.3, 0, +0.3
Formulation of the labor scheduling problem	A model (formulation)	3:DFLSP, KFLSP and NFLSP

Table 3. Summary of experimental results.

Problem set	Performance criterion	Model [b]		
		DFLSP	KFLSP	NFLSP
The complete set of 1,152 test environments	Average labor hours in the heuristically-generated schedules	2095.79	1549.88	1663.50
	NPV profit of the heuristically-generated schedules [c]	13859.39	14088.23	14167.23
	Number of test environments in which the model yielded the schedule giving the highest profit	0	308	846
20 environments randomly selected from the complete set of test environments	Number of verified-optimal solutions [d]	14	4	5
	Objective value in the best-known integer schedules [e]	1820.00	1752.23	2143.25
	Objective value in the heuristic schedules [e]	1820.00	1751.28	2146.28
	NPV profit of the best-known integer schedules [c]	11869.46	12064.45	12128.97
	NPV profit of the heuristically-generated schedules [c]	11878.30	12052.46	12131.88
	Time to find the best-known integer schedules [f]	355.42	828.25	763.71
	Time to heuristically generate schedules [f]	30.04	30.04	30.05

[a] All values are in Labor-Hour Equivalents, except schedule generation times, which are in seconds on a Pentium 90-based personal computer.

[b] DFLSP is the formulation of Dantzig (1954), which treats the employee requirements as minimum acceptable staffing levels. KFLSP is the formulation of Keith (1979), which treats the employee requirements as target staffing levels. NFLSP is the new formulation of the labor scheduling problem presented in this paper.

[c] Average weekly net-present value profit of the schedules.

[d] Number of optimal solutions obtained using the IP software during the 15-minutes-per-problem time-limit.

[e] Average objective value, in LHEs. Lower objective values are superior for DFLSP and KFLSP, while higher objective values are superior for NFLSP.

[f] Average time required to generate the labor schedules. IP-approach limited to 15 minutes per problem, simulated annealing heuristic limited to 30 seconds per problem.

Table 4. Best and worst case performance on NFLSP compared to DFLSP and KFLSP.

Best/worst	Compared to	Level of [a] AP	DV	UV	SD	DC	SP	EA	Weekly net NPV profit [b]
Best	DFLSP	2	1	2	2	1	1	3	767.01
Best	KFLSP	2	1	1	2	2	2	1	1349.26
Worst	DFLSP	2	2	2	1	2	1	1	54.80
Worst	KFLSP	3	1	2	2	1	2	3	− 107.89

[a] AP is the arrival-rate pattern, DV is the within-day arrival-rate variation, UV is the unpredictable variation in the mean daily arrival rate, SD is the mean service duration, DC is the cost of a dissatisfied customer, SP is the extra-satisfaction profit, EA is the accuracy of the cost/profit estimates.

[b] Defined as the profitability of NFLSP's schedules less the profitability of the competing model's schedules. Measured in Labor-Hour Equivalents, and averaged over the four test environments given by the combinations of the factors representing the weekly variation in the mean daily arrival rate and the waiting-time/satisfaction functional relationship.

Table 5. Marginal benefit of increasing staff sizes, by period, for the sample problem

Period		Marginal NPV benefit of increasing the number of staff by the specified number of employees (in Labor-Hour-Equivalents) [a]			
Period	MASS [b]	1	2	3	4
1	1	8.280	0.757	0.068	
2	2	9.271	1.384	0.205	0.029
3	1	14.212	1.417	0.148	0.014
4	1	8.575	0.776	0.069	
5	1	11.804	1.142	0.113	0.010

[a] Ignoring labor costs.

[b] Minimum acceptable staff size for the period – the smallest number of staff for the period who provide an aggregate service rate that exceeds the expected arrival rate.

Figure 1

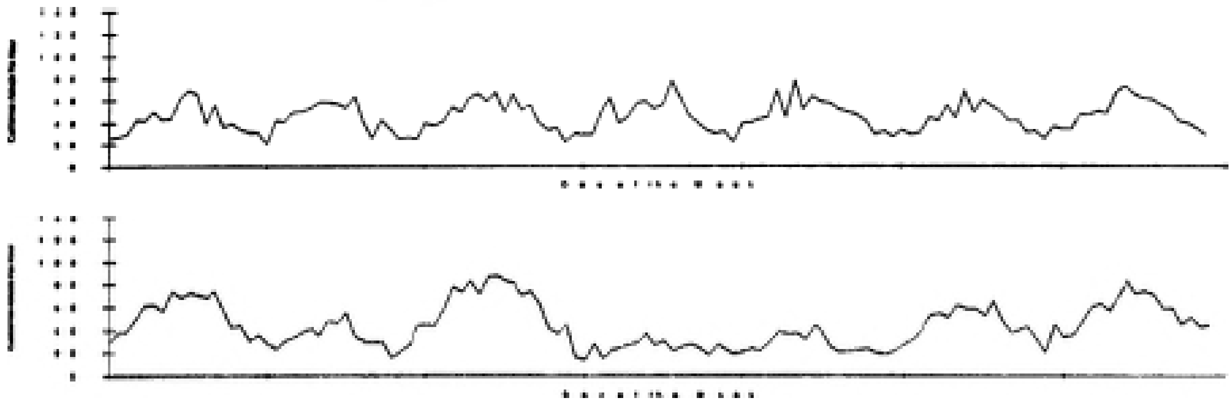

Fig. 1. Two one-week examples of simulated customer arrivals for the unimodal arrival-rate pattern with the low levels of within-day arrival-rate variation and within-week variation in the mean daily arrival rate. The top and bottom graphs show the low and high levels of unpredictable variation in the mean daily arrival rate, respectively.

Figure 2

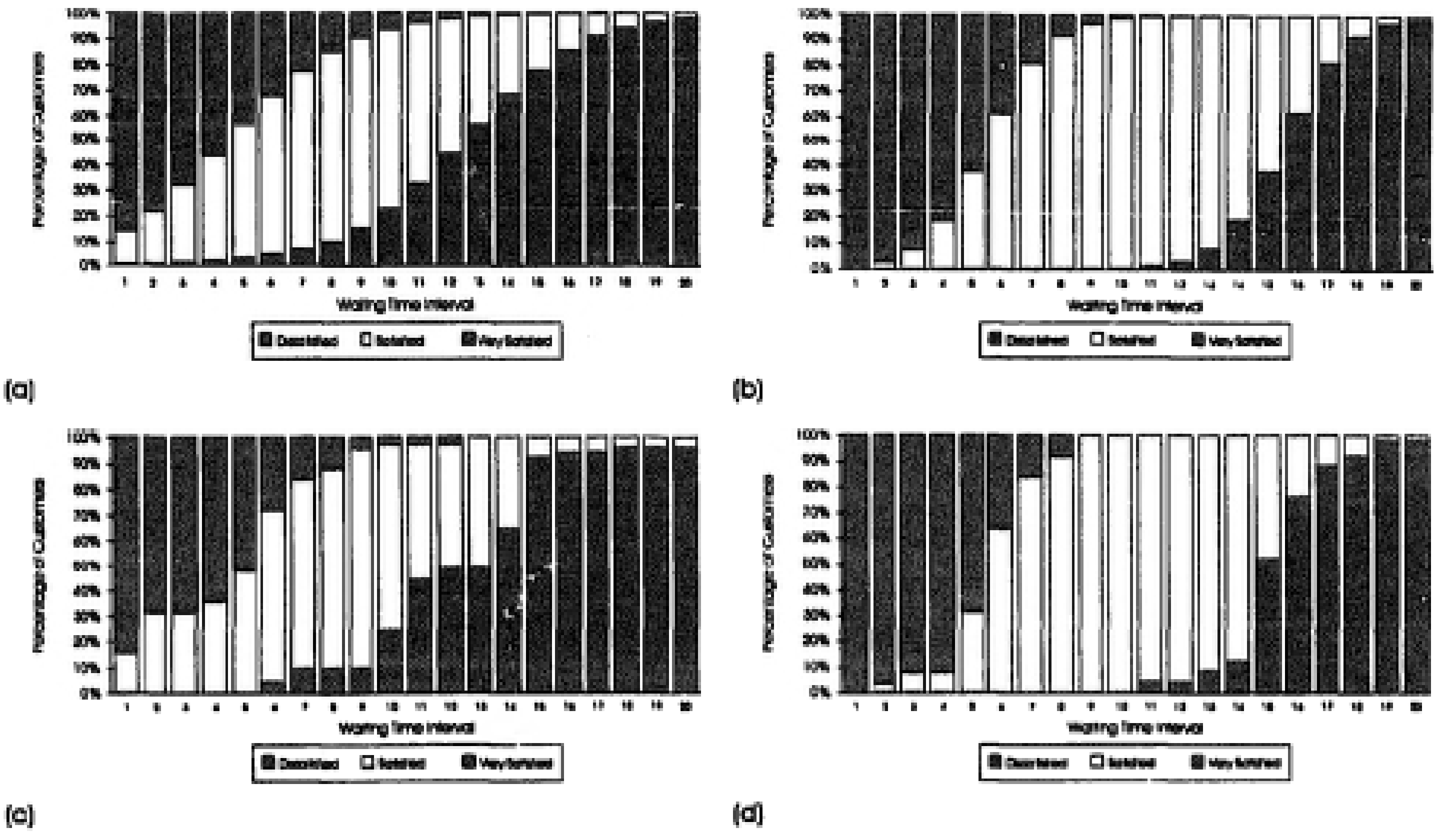

Fig. 2. The functional relationship between customer waiting time and satisfaction. The true (a) gradual and (b) rapid waiting-time/satisfaction functional relationships, and the (c) gradual and (d) rapid waiting-time/satisfaction functional relationships, determined by sampling 25 customers from each waiting time interval. All charts start at 0 minutes waiting time. For the gradual (rapid) waiting-time/satisfaction functional relationship each interval is 0.125 (0.1) minutes.

Figure 3

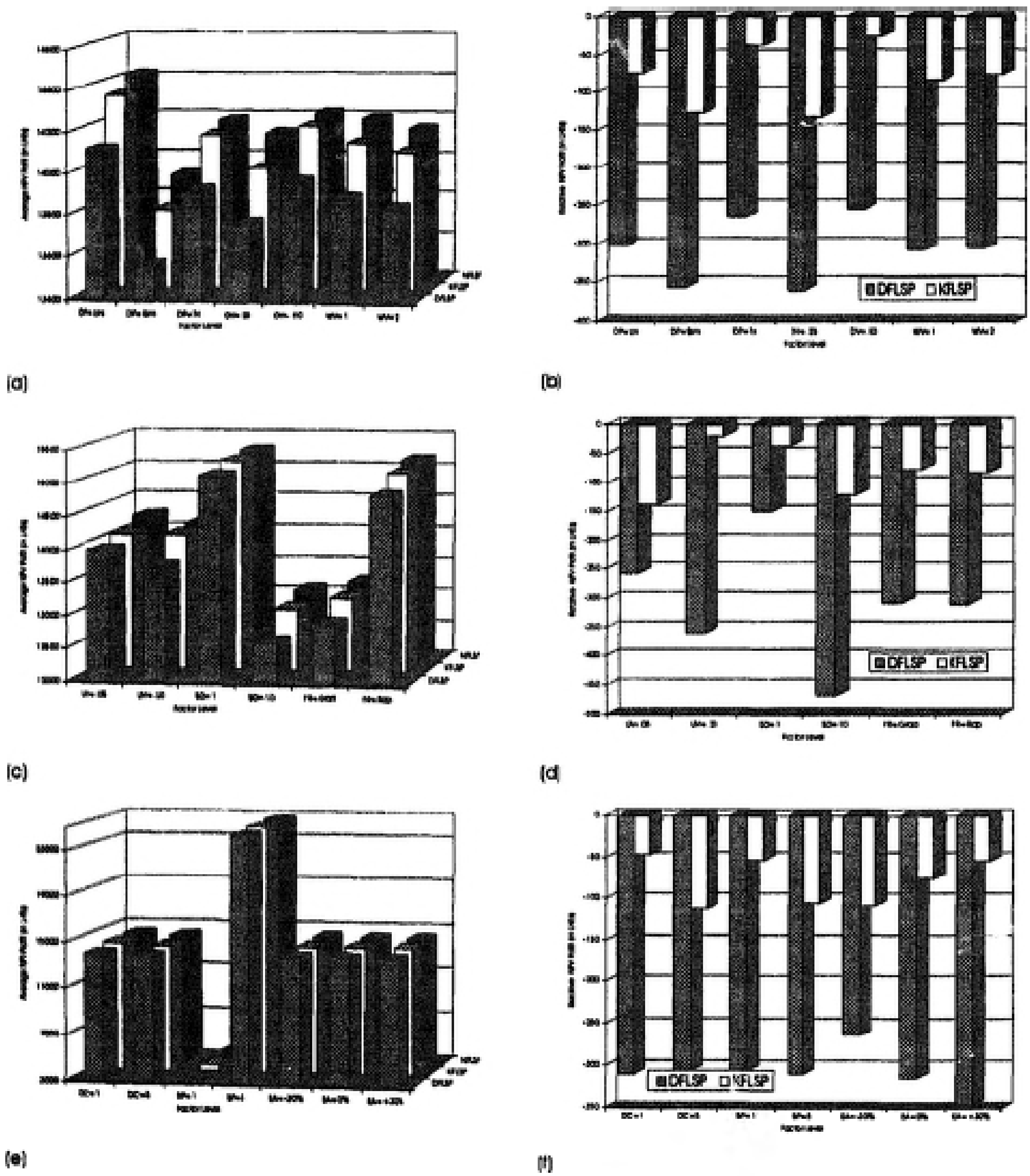

Fig. 3. Average actual and relative (to NFLSP) weekly NPV profits, measured in LHEs, by model formulation, for (a) and (b) the levels of arrival-rate variation (DP), within-day variation in the arrival rate (DV), and within-week variation in the mean daily arrival rate (WV); for (c) and (d) the levels of unpredictable variation in the mean daily arrival rate (UV), mean service duration (SD), and waiting-time/satisfaction functional relationship (FR); and for (e) and (f) the levels of dissatisfaction cost (DC), extra-satisfaction profit (SP), and the accuracy of the estimates of the dissatisfaction cost and extra-satisfaction profit (EA).

References

Bailey, J., 1985. "Integrated days off and shift personnel scheduling", Computers and Industrial Engineering, vol. 9, no. 4, 395-404.

Bailey, J. and J. Field, 1985. “Personnel scheduling with flexshift models", Journal of Operations Management, vol. 5, no. 3, .327-338.

Baker, K.R., 1976. "Workforce allocation in cyclical scheduling problems: a survey", Operational Research Quarterly, vol. 27, no. 1, 155-167.

Baker, K.R., T.B. Crabill and M.J. Magazine, 1973. “An optimal procedure for allocating manpower with cyclic requirements", AUF Transactions, vol. 5, no. 2, 119-126.

Bartholdi, J.J., 1981. “A guaranteed-accuracy round-off algorithm for cyclic scheduling and set covering", Operations Research, vol. 29, no. 3, 501-510.

Bechtold, S.E. and L.W. Jacobs, 1990. "Implicit optimal modeling of flexible break assignments in labor staffing decisions for service operations", Management Science, vol. 36, no. 11, 1339-1351.

Bechtold, S.E. and L.W. Jacobs, 1991. “Improvement of labor utilization in shift scheduling for services with implicit optimal modeling'*, International Journal of Operations and Production Management, vol. 11, no. 2, 54-69.

Bechtold, S.E. and M.J. Showalter, 1985. “Simple manpower scheduling methods for managers", Production and Inventory Management, vol. 26, no. 3, 116-132.

Bechtold, S.E. and M.J. Showalter, 1987. “A methodology for labor scheduling in a service operating system", Decision Sciences, vol. 18, no. 1,89-107.

Bechtold, S.E., M.J. Brusco and M.J, Showalter, 1991. “A comparative evaluation of labor tour scheduling methods**, Decision Sciences, vol. 22, no. 4, 683-699, Brooke, A., D. Kendrick and A, Meeraus, 1992. GAMS, Release 2.25, A User's Guide, The Scientific Press, South San Francisco, CA.

Brusco, M.J. and L.W. Jacobs, 1993. “A simulated annealing approach to the cyclic staff-scheduling problem", Naval Research Logistics, vol. 40, 69-84.

Buffa, E.S., M.J. Cosgrove and B.J. Luce, 1976. "An integrated work shift scheduling system", Decision Sciences, vol. 7, no. 4, 620-630.

Dantzig, Ü.B., 1954. "A comment on Edie's ‘Traffic delays at toll booths' ", Operations Research, vol. 2, no. 3, 339-341.

Davis, M., 1991. “How long should a customer wait for service?", Decision Sciences, vol. 22, no. 2, 421-434.

Easton, F.F. and D.F. Rossin, 1991a. “Sufficient working subsets for the tour scheduling problem”, Management Science, vol. 37. no. 11, 1441-1451.

Easton, F.F. and D.F. Rossin, 1991b. “Equivalent alternate solutions for the tour scheduling problem", Decision Sciences, vol. 22, no, 5, 985-1007.

Gaballu, A, and W, Pearce, 1979, “Telephone sales manpower planning at Qantas", Interfaces, vol. 9, no. 3, 1-9.

Henderson, W.B, and W.L, Berry, 1976. "Heuristic methods for telephone operator shift scheduling: an experimental analysis". Management Science, vol. 22, no. 12, 1372-1380.

Henderson, W.B, and Wu Berry, 1977. “Determining optimal shift schedules for telephone traffic exchange operators", Decision Sciences, vol. 8, no. 1, 239-255,

IBM Corporation, 1991, Optimization Subroutine Library, Release 2, IBM Corporation, Kingston, NY.

Jacobs, L.W. and S.E. Bechtold, 1993. "Labor utilization effects of labor scheduling flexibility alternatives in a tour scheduling environment", Decision Sciences, vol. 24, no. 1, 148-166.

Keith, E.G., 1979, "Operator scheduling", A1IE Transactions, vol. 11, no, l, 37-41.

Krajewski, L.J., L.P. Ritzman and P. McKensie, 1980, “Shift scheduling in banking operations: a case application". Interfaces, vol. 10, no, 2, 1-8,

Li. C, E.P. Robinson Jr and V.A. Mabert, 1991. “An evaluation of tour scheduling heuristics with differences in employee productivity and cost", Decision Sciences, vol. 22, no. 4. 700-718,

Loucks, J.S. and F.R. Jacobs, 1991. “Tour scheduling and task assignment of a heterogeneous work force: a heuristic approach", Decision Sciences, vol. 22, no. 4, 719-738.

Mabert, V.A., 1979. “A case study of encoder shift scheduling under uncertainty", Management Science, vol. 25, no. 7, 623-631.

Mabert, V.A. and M.J. Showalter, 1990. “Measuring the impact of part-time workers in service organization", Journal of Operations Management, vol. 9, no. 2, 209-229.

Mabert, V.A. and CA. Watts, 1982. “A simulation analysis of tour-shift construction procedures", Management Science, vol. 28, no. 5, 520-532.

McGinnis, L.F., W.D. Culver and R.H. Deane, 1978, "One- and two-phase heuristics for workforce scheduling", Computers and industrial Engineering, vol. 2, no. 1, 7-15.

Moondra, S.L., 1976. "An L.P. model for work force scheduling for banks", Journal of Bank Research, vol. 7, no. 4, 299-301.

Morris, J.G, and M.J. Showalter, 1983. "Simple approaches to shift, days-off and tour scheduling problems", Management Science, vol. 29, no. 8, 942-50.

Quinn, P., B. Andrews and H. Parsons, 1991. “Allocating telecommunications resources at L.L. Bean”, Interfaces, vol. 21, no. 1, 75-91.

Segal, M., 1974. “The operator-scheduling problem: a network- flow approach'*, Operations Research, vol. 22, no. 4,808-823.

Showalter, M.J. and V.A. Mabert, 1988. “An evaluation of a full-/part-time tour scheduling methodology", International Journal of Operations and Production Management, vol. 8, no. 7, 54-71.

Taylor, P.E. and S.J. Huxley, 1989, “A break from tradition for the San Francisco police: patrol officer scheduling using an optimization-based decision support system”, Interfaces, vol. 19, no. 1, 4-24.

Thompson, G.M., 1990. “Shift scheduling when employees have limited availability: an L.P. approach", Journal of Operations Management, vol. 9, no. 3, 352-370.

Thompson, G.M., 1992. "Improving the utilization of front-line service delivery system personnel", Decision Sciences, vol. 23, no. 5, 1072-1098,

Thompson, G.M., 1993. “Accounting for the multi-period impact of service when determining employee requirements for labor scheduling", Journal of Operations Management, vol. 11, no. 3, 269-287.