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Diversification Benefits of REIT Preferred and Common Stocks: A Long-Run Empirical Analysis

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Keywords

Cornell, diversification, real estate investment trust (REIT), Modern Portfolio Theory, preferred stock

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We study the diversification benefits of REIT preferred and common stocks. Taking the view of a long run investor, we conduct our analysis using data from 1992 to 2012. We examine optimal mean-variance portfolios of an investor given access to different classes of assets and establish five main results. First, preferred stock provides significant diversification benefits to all equity investors. Second, preferred stock appears to be a bond substitute. Third, preferred stock provides a venue for risk reduction for constrained investors who have access to bonds. Fourth, REITs provide an important value dimension to investors. Finally, REITs allow long only investors the ability to form higher total return portfolios than they otherwise would have been able to attain.

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1 Introduction

Real Estate Investment Trusts use a variety of venues to raise capital. They mortgage their properties, raise debt in the public capital market, and issue common stock and partnership units. They also rely on preferred stock. According to NAREIT, from 1992 through 2012 equity REITs raised $52.4 billion through preferred issuance, a number larger than the $41.3 billion raised in REIT IPOs, and approximately one fifth of all the public equity issued by REITs.[2]

In this paper we call attention to REIT preferred stocks. We take an investor view and focus on the performance of REIT preferred stock as an asset class. In equilibrium, for REITs to be able to raise capital through preferred issuance, this asset class needs to offer attractive risk-return characteristics for investor portfolio formation and diversification.

Preferred stock has received some attention in the academic literature. However, these papers tend to focus on either why firms choose to issue preferred stock over other classes of securities,[3] the role of preferred stock in the capital structure,[4] or on the pricing of preferred stock.[5] The issue we examine in this paper is why investors might choose to hold these securities.

Preferred stock is a "hybrid" security, occupying a place between debt and common equity. In fact, classification of the preferred--is it debt? is it equity?--has been an interesting question for some time. In their book "Security Analysis" that survived six editions to become a classic, Benjamin Graham and David Dodd in 1934 raise what they call "*objections to the conventional grouping: preferred stock grouped with common.*" In their view, "*while this approach is hallowed by tradition, it is open to several serious objections. Of these the most obvious is that it places preferred stocks with common stocks, whereas, so far as investment practice is concerned, the former undoubtedly belong with bonds. The typical or standard preferred stock is bought for fixed income and safety of principal.*" Is Graham and Dodd's conjecture correct? Do REIT preferred offer investors a dimension similar to bonds, or are preferred stocks an equity asset class?[6]

We study the risk and return characteristics of REIT preferred and common stock within the context of Modern Portfolio Theory and address diversification benefits of these asset classes from the point of view of investment portfolios. A closely related question, then, is whether REIT preferred stocks have attractive risk and return characteristics. Turning to "Security Analysis," we find that Graham and Dodd begin their chapter "The Theory of Preferred Stocks" with quite a negative observation regarding the preferred: "*That the typical preferred stock represents an unattractive form of investment contract is hardly open to question. On the one hand, its principal value and income return are both limited; on the other hand, the owner has no fixed, enforceable claim to payment of either*

[2] Lee and Johnson (2009) report that 20% of NYSE firms, 15% of AMEX firms and 175 of NASDAQ firms have preferred stock in their capital structure; Yaman (2011) reports that that slightly more than one-third of all non-IPO equity raised by U.S. firms was preferred equity over the 1985-1999 timeframe, with the U.S. public market for preferred stock reaching $193billion in 2005. Kallberg, Liu and Villupuran (2013) report that U.S. firms used preferred equity to raise over 47% of all equity capital from 1999 – 2005, including IPOs; further preferred equities were almost 60% of all non-IPO equity raised during this period.

[3] See, Boudry, Kallberg, and Liu (2010).

[4] See, for example, Ott, Riddiough and Yi (2005), and Heinkel and Zechner (1990).

[5] See, for example, Bildersee (1973), Sorensen and Hawkins (1981), Emanuel (1983), and Ferreira, Spivey, and Edwards (1992).

[6] It appears even firms themselves are uncertain how to classify preferred stock. In a recent study, Kallberg, Liu and Villupuran (2013) find 28% of new preferred issues classified as debt, 33% of new issues classified as equity, and 39% of new issues classified as a hybrid instrument.

principal or income." While the authors believe that preferred stocks of high quality issuers selling at a discount may present attractive risk-return characteristics, they are certainly not enthusiastic about the average preferred stock. Are preferred stocks, on average, an attractive asset class that warrants a non-trivial allocation in a portfolio?

To examine the potential benefits that preferred stock may or may not provide to an investor, we examine the optimal mean-variance portfolios of an investor given access to different classes of assets. To make our results as general as possible, we give our investor access to global and US large cap stock indices, US small and mid-cap equity portfolios, US small and mid-cap growth and value portfolios, investment grade and high yield bond indices, an equity REIT index, and a REIT preferred stock index. Taking the view of a long run investor, we conduct our analysis using data from the largest time series available, with our data spanning the period November 1992 to November 2012.

We find five main results. First, preferred stock provides significant diversification benefit to an all equity investor. With unconstraint portfolio formation, access to preferred stock increases the investor's Sharpe Ratio by between 7.92% and 63.39%. The wide range in Sharpe Ratio improvement is due to the potential equity assets that the investor has access to. Our most limited investor has access only to the S&P 500 and the World Ex-US indices and enjoys the largest increase in Sharpe Ratio, while the least limited investor has access to all the equity assets described above and receives the smallest benefit. In constrained portfolio formation (no short sales,) we find a similar result: access to preferred stock increases the investor's Sharpe Ratio by between 61.82% and 25.03%.

Second, preferred stock appears to behave like a bond substitute. When the investor is given access to bond indices, the increase in Sharpe Ratio due to the inclusion of preferred stocks is markedly smaller, with improvements between 0.28% and 3.06% for an unconstrained investor, and between 1.73% and 3.35% for a constrained (no short sales) investor. So it appears from an investor's perspective that preferred stock is debt rather than equity. The intuition of Graham and Dodd who in their 1934 book proposed to place preferred stocks with bonds is confirmed in the Modern Portfolio Theory analysis.

Third, preferred stock provides an investor with a venue for risk reduction. While the improvements in Sharpe Ratio due to preferred stock for a constrained investor who has access to bonds are small, access to preferred stock reduces the variance of the optimal portfolio. This is especially important for investors with relatively high risk aversion.

Fourth, for an all equity investor, access to REIT common shares provides an important value dimension to their portfolio. When value indices are not a part of the investment opportunities set, inclusion of REIT common stock results in a substantial improvement in the risk-return tradeoff. Sharpe Ratios improve by between 6.83% and 28.62% for an unconstrained investor, and 8.87% and 26.53% for a constrained (no short sales) investor. These improvements nearly completely disappear when an investor has access to mid-cap value or small-cap value indices. Papers in the prior literature tend to allow an investor access to size portfolios, but not value/growth portfolios.[7] Our results indicate that the benefit of REITs in a mixed asset portfolio is very sensitive to the omission or inclusion of value portfolios.

Finally, although Sharpe Ratio improvement due to REIT common shares are low for investors who have access to value portfolios, REIT common shares do provide one distinct benefit. Under the realistic scenario

[7] See, for example, Sa-Aadu, Shilling, and Tiwari (2010).

for a long term investor of no short sales, REIT common shares are crucial for achieving portfolios with high expected returns. The inclusion of REIT common shares allows investors to form total return portfolios they would not have been able to otherwise form.

Overall our results are relevant to both academics and practitioners. First, they suggest that investors, especially those with high risk aversion who focus on equity portfolios, would be well served by investing in REIT preferred stocks. Doing so would be beneficial to the risk adjusted performance of their portfolios. Second, low risk aversion investors, especially those with long only portfolios, can use REITs to attain higher total return portfolios. Third, both academics and investors should be cognizant of the value nature of REITs. From an academic perspective it means that benchmarking, even based on size portfolios, is likely to provide misleading inferences about REIT stock performance. For investors, REIT common shares are likely to provide the most significant diversification benefits for investors who do not already have a mid-cap value or small-cap value stocks in their portfolios.

The remainder of this study is organized as follows. Section 2 provides a review of the literature, while Section 3 describes our empirical methodology and data. Section 4 presents our results, and Section 5 provides our conclusions.

2 Literature Review

The literature that this paper is most closely related to is the literature examining the role of real estate in mixed asset portfolios. This literature applies the Modern Portfolio Theory framework of Markowitz (1952) and Merton (1972) to examine the benefit of holding real estate (both direct and securitized in the form of REITs) in mixed asset portfolios.

Beginning in the early 1980's a series of papers demonstrates that positive allocations to real estate produce superior risk-adjusted returns to portfolios that omit real estate, especially for risk sensitive investors. These studies rely on the NCREIF index of real estate returns, a database of private real estate returns or an index of returns from comingled real estate funds. Using this data, Miles and McCue (1984) find that real estate acts as an inflation hedge and that there are other intriguing properties of real estate returns, notably the low correlation with stocks and bonds; Ross and Zissler (1991) estimate allocations in the range of 9 to 13 percent while Folger (1984) makes the case for a 20% allocation using these data. Ibbotson and Siegel (1984) confirm the findings of Miles and McCue (1984) and Folger (1984), but also show that the pricing of real estate reacts to very different stimuli than stocks and bonds; real estate prices are shown to be sensitive to residual risk and factors such as taxes, marketability and information costs that are not captured in the traditional measure of risk as represented by market or beta risk.

An early critique of these studies is that they rely on appraisal based returns indexes, which are known to understate the volatility of real estate returns, called appraisal smoothing. Corgel and deRoos (1999) correct for the understated volatility and find significant allocations to real estate in the ranges found by previous authors. While the correction for appraisal smoothing adds volatility to real estate returns, thus reducing the allocation, the correction also makes real estate less correlated with stocks and bonds, increasing the allocation. Kallberg, Liu, and Greig (1996) find support for allocations of approximately 9% to real estate, but in addition, find a "size effect," that is, small properties tend to bring much larger diversification benefits to mixed asset portfolios than large properties due to their lower correlation with other financial assets while maintaining a similar risk/return profile as large properties.

Starting in the mid-1990's, real estate researchers added REITs to the NCREIF return series, allowing an exploration of both public market real estate returns in addition to the private market returns. Gyourko and Nelling (1996) explore the impact of different property types and location in real estate portfolio allocation and find that diversification by location and property type is independent to market based diversification measures. Kallberg, Liu, and Trzcinka (2000) find that REIT mutual funds have persistent positive net alphas, a significant finding. Combining the conventional wisdom that real estate provides diversification and an inflation hedge, Lin and Yung (2006) find that REIT returns affect REIT equity market flows, not vice versa, meaning that the market demand for REIT equities is closer to horizontal than downward sloping.

More recently, Sa-Aadu, Shilling, and Tiwari (2010) examine the performance of real estate in mixed asset portfolios in a conditional asset pricing framework and find that real estate is one of two asset classes that deliver portfolio gains during bad times; that is, when investors really care about returns. They argue that investors may underweight real estate due to this misunderstanding of real estate return behavior in bad times.

3 Data and Methodology

3.1 Theoretical Framework of Modern Portfolio Theory

To examine the potential benefits of preferred stock to investors, we employ the framework of modern portfolio theory developed by Markowitz (1952) and Merton (1972). Let there be N risky assets available to the investors. The investment opportunity set is described by the vector of expected returns on the N assets, $\bar{z}$, and by $\hat{\sum}$, the covariance matrix. Modern portfolio theory assumes that the investors' preferences can be represented by a utility function defined over mean (the expected return) and variance of a portfolio's return. The assumption is that investors favor higher means and smaller variances. Minimum-variance portfolios are portfolios that have the smallest variance for every level of expected returns. In the absence of a risk-less asset the minimum variance portfolio with expected return μ is the solution $w(\mu)$ to

$$\min \frac{1}{2} w' \hat{\sum} w \tag{1}$$

$$\textit{Subject to} \quad 1'w = 1,$$

$$\bar{z}'w = \mu$$

Vector w is the vector of weights of risky securities in the portfolio. When no positivity constraints of the form $w_i \geq 0$ are imposed, unrestricted short sales are permitted. With no short sale restrictions and no risk-free asset available, in mean-standard deviation space, with mean return on the y-axis and standard deviation on the x-axis, the set of minimum variance portfolios is a hyperbola, as illustrated in Figure 3. When a risk-less asset is available a Capital Allocation Line (CAL) obtains. It is a straight line which intersects the y-axis at the risk-free rate and is tangent to the minimum variance frontier constructed from risky assets. The Capital Asset Pricing Model identifies the tangency point as the market portfolio. The Sharpe Ratio, which equals the slope of the CAL, characterizes the risk-return trade-off of the market portfolio and is computed as

$$S_p = \frac{r_p - r_f}{\sigma_p}, \tag{2}$$

where r_p is the expected return on the market portfolio, r_f is the risk-free interest rate, and σ_p is the standard deviation of the return on the market portfolio. Portfolios with higher Sharpe ratios offer more attractive risk-return trade-off.

Investors frequently face short-sales constraints. When no risk-free asset is available, the optimization problem (1) is modified by adding short sales constraints $w_i \geq 0$ for all assets, $i = 1,\ldots, N$. Generally, with positivity constraints the frontier is not a hyperbola.[8] If a risk-free asset is available, it is still possible to draw a straight line with the intercept equal to the risk-free rate tangent to the minimum variance frontier. The tangency point is the market portfolio in this case, and its Sharpe ratio (the slope of the line) characterizes the risk-return trade-off that the investors in the economy face.

To evaluate the role of REIT preferred stocks and REIT common stocks we first construct minimum variance portfolios without REIT preferred or REIT common shares. We then include preferred shares only, REIT common shares only, and both REIT preferred and REIT common shares in the investment opportunity set and construct minimum variance portfolios again. We follow this procedure to quantify the improvement in the risk-return trade-off due to the inclusion of these asset classes. We consider both the unconstrained case and the constrained case (short sales constraints are added).

3.2 Data

To provide some evidence on the magnitude of REIT preferred issuances, we collect REIT issuance data for stocks (both SEO and IPO), preferred stock, and public debt from NAREIT. Table 1 reports this data. Over our sample period from 1992 to 2012, REITs issued $493.3 billion of public capital. Of this, $52.39 billion was preferred stock, $218.2b was public debt, $41.31b was IPO proceeds, and $181.46b was secondary equity issuances. Preferred stock represents approximately 11% of all public proceeds raised, or 19% of all public equity raised. In fact, REITs raised more proceeds from preferred stock issuances than through IPOs.

We collect monthly returns from November 1992 to November 2012 from four sources. Datastream provides returns on the Barclays Investment Grade Corporate Bond Index, the Barclays High Yield Corporate Bond Index, the MSCI World Index, the MSCI World Ex-US index, the Russell 2000 index, the Russell 2000 growth index, the Russell 2000 value index, the Russell Mid Cap index, the Russell Mid Cap growth index, and the Russell Mid Cap value index. From SNL we collect the SNL US Equity REIT index. MSCI provides the MSCI REIT Preferred Index.[9] From CRSP we obtain returns on the 30-day T-Bill and the returns on the S&P500.

Table 2 reports descriptive statistics for the indices used in our analysis. Panel A reports means and standard deviations, while Panel B reports correlations. The average annualized return for REIT preferred stock over

[8] Dybvig (1984). Ukhov (2006) studies changes in the mean-variance frontier caused by changes in the asset set.

[9] We are able to obtain the constituents of the MSCI REIT Preferred Index for 2005 on. An examination of these constituents shows that the index is extremely broad based. There are over 100 constituents in the index and no individual constituent is more than 2% of the index. We are also able to obtain daily REIT preferred stock prices and trading volume from 2000 on and observe that most REIT preferred stock trades at a daily frequency. This suggests that although they are less liquid than there common share counterparts, they are liquid enough for an investor to be able to form portfolios.

the sample period was 10.3% with a standard deviation of 11.4%. This compares to an average return of 12.9% for REIT common stock, which was the asset class with the highest average return in our sample. The S&P 500 also performed well over the sample period with a mean return of 9.2%. As would be expected over a long sample, investment grade bonds had the lowest average return of 7.1%.

Turning to the correlations in Panel B, we observe some noticeable characteristics. First, preferred stock is most highly correlated with high yield bonds with a 68% correlation, but also shares significant co-movement with REITs. Second, an examination of the correlation between REIT common stock and the other equity indices shows that REITs are more highly correlated with small and mid-cap stocks than large cap, but also that REITs appear to have a significant value component to them.[10] For both the mid-cap and Russell 2000 universes, REITs are more highly correlated with value stocks than growth stocks.[11] Finally, the asset class that appears to be least correlated with the others is investment grade bonds. It has high correlations with only the high yield bond index and REIT preferred stock index.

Figure 1 reports the current value of a dollar invested in each asset class in 1992. Some clear patterns emerge regarding the time series behavior of the asset classes. First, consistent with the average returns observed in Table 1, REIT common stock was the best performing asset class by the end of the sample period. Notice however, that this was not always the case. Prior to 2000 REIT common shares were nearly always the worst performing equity index and up to that time had a total return nearly identical to high yield corporate bonds. In fact prior to 2002, REIT common stock and preferred stock had remarkably similar performance. Second, from 2002 to 2007 we observe a marked increase in the performance of midcap, small and midcap value, and REIT common shares. The close relationship between REIT common shares and midcap value stocks become quite apparent in Figure 2 when we plot just the REIT common and preferred shares, the S&P 500, and the mid cap value index. Finally, the effects of the global financial crisis, subsequent recession, and recovery are evident in the latter part of the sample. We observe all risky asset classes declining significantly during the crisis and rebounding during the recovery. This V-shaped pattern being quite pronounced in both the REIT common and preferred shares.

4 Results

4.1 Equity Asset Classes

We begin our minimum-variance analysis by using all eleven equity asset classes, including S&P 500, REIT common shares, REIT preferred shares, the world market index, the aggregate mid-cap index and mid-cap value and growth indices, Russell 2000 index, Russell 2000 value and growth, and world ex-US. This is our most inclusive equity investment opportunity set. Data from Table 2 (expected returns, standard deviations and correlations) is used to construct four mean-variance frontiers shown in Figure 3: a frontier with all equity asset classes, including preferred shares and REITs (black); a frontier that includes preferred shares but does not include REITs (blue); a frontier that includes REITs but does not include preferred shares (red); and

[10] See Ambrose, Lee, and Peek (2007) for a discussion of REIT co-movement after index inclusions.

[11] Notice that this is unlikely to be driven by REITs being components of midcap and small cap indices for two reasons. First, these are very broad based indices. Second, the Russell 2000 value index has a 97% correlation with the small/value Fama French portfolio that by construction does not include REITs. The REIT common index also has a correlation of 73% with the small/value portfolio. In this sense it appears that REITs are highly correlated with small and midcap value indices not because they are part of those indices, but because they tend to be small and midcap value stocks. We use the Russell indices in our analysis instead of Fama French portfolios simply because an investor is far more likely to hold a Russell index than a portfolio of stocks they created with size and book to market sorts.

a frontier that excludes both preferred shares and REITs (green). No short sale restrictions are imposed. We refer to this case as "unconstrained." The figure illustrates a substantial improvement in the risk-return trade-off resulting from the inclusion of preferred stocks. The two frontiers (black and blue) constructed with the REIT preferred stock index are located higher and to the left compared to the two frontiers that do not include preferred stock, indicating that any level of expected return, μ, can be attained with less risk, σ, when preferred stocks are used.

While unconstrained mean-variance analysis represents a classical approach to the portfolio problem, the optimization analysis frequently results in negative weights assigned to several asset classes. In practice investors frequently face no-short-sale constraints and are interested in the performance of long-only portfolios. Therefore, a potentially better way to judge the contribution of different asset classes to portfolio diversification is through constrained optimization analysis, where portfolio weights are constrained to be non-negative (no short sales.)

Figure 4 is similar to Figure 3, except that no short sales are allowed -- investments in all assets are constrained to be positive. The green curve shows the frontier when preferred stock and REITs are excluded from the investment opportunities set. Inclusion of preferred stocks and REITs clearly improves risk-return characteristics of the frontier. When preferred stocks are included (but REITs are not), the constrained frontier shifts strongly to the left and portfolios with low variances become attainable (blue curve). At the same, inclusion of REITs (but not preferred stock) makes portfolios with high expected returns (and high risk) attainable (shown in red.) Investors with high degree of risk aversion will find that preferred stocks offer significant diversification benefits and inclusion of preferred stocks dramatically improves the performance of long-only equity portfolios. Investors with relatively low risk aversion seeking portfolios with higher risk (and return) will find the inclusion of REITs to be crucial.

It is important to point out that the positivity constraints used in the analysis are consistent with real limitations to shorting broad asset classes. Although short-selling is certainly practiced with respect to individual securities, it is unlikely that it would be used by a long-term investor to maintain an optimal position in terms of risk and return of the portfolio.

To quantify these observations Table 3 shows the characteristics of the market (tangency) portfolios created with different mixes of equity asset classes. Both the unconstrained and constrained cases are shown. Panel A corresponds to Figures 3 and 4. In the unconstrained case, the market portfolio constructed without preferred stock or REITs has an expected return of 1.767% per month, monthly standard deviation of 6.168% and a Sharpe Ratio of 0.2451 (the corresponding annualized Sharpe Ratio is 0.849).[12] When preferred stocks and REITs are added, the Sharpe Ratio improves by nearly 8% to equal 0.2646 (annualized 0.917). In the constrained case, the market portfolio constructed without preferred stock or REITs has an expected return of 0.984% per month, monthly standard deviation of 4.616% and Sharpe Ratio of 0.1578 (annualized Sharpe Ratio is 0.547). When preferred stocks and REITs are added, the Sharpe Ratio improves by 25% to 0.1973 (0.683 annualized).[13] Also note the diversification benefits of the preferred stock. When only preferred

[12] To obtain annualized returns monthly returns should be multiplied by 12; to obtained annualized standard deviation, monthly returns should be multiplied by square root of 12. Sharpe ratio is defined as $(r - r_f)/\sigma$. Therefore, the annualized Sharper Ratio is obtained from Sharpe Ratio based on monthly numbers by multiplying it by $\sqrt{12}$.

[13] The magnitude of these Sharpe ratios is similar to the numbers reported in Goetzmann and Ukhov (2006) study of UK international investment. Using five asset classes (domestic common, domestic preferred, domestic debt, foreign common and foreign debt), they show that in unconstrained case the annualized Sharpe ratio increases from 0.70 in

(without REIT common) is added to the investment opportunity set in the constrained case, the Sharpe Ratio increases by 25% relative to the base case. Investors in equity asset classes reap economically significant diversification benefits from the inclusion of preferred stocks in their investment opportunity set. The economic significance is similar in magnitude to the benefits from diversifying internationally by UK equity investors as reported in Goetzmann and Ukhov (2006).

The all-equity investment opportunity set includes S&P 500, MSCI World Index and MSCI World ex-US Index. In the remainder of this section we exclude MSCI World Index because we include both S&P 500 and MSCI World ex-US. The efficient frontiers with MSCI World excluded look remarkably similar to Figures 3 and 4. The impact on the Sharpe Ratio of the addition of preferred stock to the investment opportunity set is similar to that reported above (reported in Table 3, Panel 2).

4.11 The Value Dimension

Next, we investigate the role that REIT preferred stock and REIT common stock play relative to the value dimension. The well-documented value premium in empirical asset pricing implies that value stocks have attractive risk-adjusted returns.[14] An important question to investigate, then, is to what extent do REIT common and preferred stocks contribute to improved risk-return trade-off relative to the characteristics of value stocks? We perform several experiments to address this question.

First, we compare the contribution of preferred stock and REITs to diversification relative to the inclusion (or exclusion) of Mid-Cap Value and Russell 2000 Value indices. Table 3, Panel 3 reports optimal portfolio characteristics when the aggregate Mid-Cap and Russell 2000 indices are included, but the corresponding value and growth indices are excluded (we exclude Mid-Cap Value, Mid-Cap Growth, Russell 2000 Value and Russell 2000 Growth). The results can be contrasted with Table 3, Panel 4 that describes the case when value and growth indices *are* included (Mid-Cap Value, Mid-Cap Growth, Russell 2000 Value and Russell 2000 Growth), but the aggregate indices (Mid-Cap and Russell 2000) are not. When only the aggregate indices are included preferred stock and REITs have a large impact on the risk-return tradeoff of the optimal portfolios. Consider the unconstrained case first. Inclusion of both preferred stock and REITs yields improvement in the Sharpe Ratio of 19.58% relative to the benchmark case. Inclusion of preferred stock only, improves the Sharpe Ratio by 19.37%, and inclusion of REITs (but not preferred stock,) improves the Sharpe Ratio by nearly 7%. The improvement is smaller when the set of investment opportunities already includes Mid-Cap Value and Russell 2000 Value indices. When value indices are included, addition of both preferred stock and REITs yields improvement in the Sharpe Ratio of 10.5%, while inclusion of preferred stock (and not REITs) yields a Sharpe Ratio improvement of 10.05%. Inclusion of REITs only, yields a small improvement in the Sharpe ratio (0.46%) when value indices are already included.

The results for the constrained case are similar. When value indices are not included, adding preferred stock and REITs improves the Sharpe Ratio by 35% compared to an improvement of 25% in the case when value and growth indices are included (the numbers for the preferred stock only case are similar). REITs plays a more important role in the absence of value indices. When only the aggregate Mid-Cap and Russell 2000

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domestic-assets-only case to 0.85 when foreign asset classes are added. In the constrained case, the improvement is from 0.67 to 0.81, or 20.90%. Blackburn et. al. (2009) report Sharpe ratios for a variety of size and value portfolios, which are similar in magnitude.

[14] See Blackburn et. al. (2009) and references therein.

indices are included, addition of REITs improves the Sharpe Ratio by 8.87%, relative to a 2.47% improvement when value and growth indices are a part of the set.

Second, we compare contribution of preferred stock and REITs to diversification relative to the inclusion (or exclusion) of Mid-Cap Value and Growth indices when all three Russell 2000 indices are excluded. The results are reported in Table 3, Panels 6 and 7. When only the aggregate Mid-Cap index is included, addition of both preferred stock and REITs improves the Sharpe Ratio by 29.3%, compared to a 20.76% improvement when Mid-Cap Value and Growth are a part of the set. Inclusion of REITs only, yields an improvement of 6.82% in the Mid-Cap aggregate case (nearly 9% in the constrained case,) versus 1.1% (2.5% under constraints) improvement in the case of Mid-Cap Value and Growth. REITs play a more important role when the value dimension is not a part of the initial set.

Finally, in the third experiment, Mid-Cap, Mid-Cap Value and Growth indices are excluded. Russell 2000 and Russell 2000 Value and Growth indices are used in different combinations. Figure 5 illustrates this experiment for the unconstrained case. Panel A shows minimum-variance frontiers for the case when the aggregate Russell 2000 index is excluded, but Russell 2000 Value and Growth indices are included. Inclusion of preferred stock only (blue curve,) results in a strong shift of the efficient frontier to the left, nearly matching the frontier that includes both preferred stocks and REITs (black curve). Preferred stocks play an important role in risk reduction. At the same time, in the presence of Russell 2000 Value index, inclusion of REITs only (no preferred stock,) causes a relatively small expansion of the frontier (red) relative to the benchmark case of no real estate (green).

Panel B shows minimum-variance frontiers for the case when only the aggregate Russell 2000 index is included, but Russell 2000 Value and Growth indices are not. We observe two effects. First, here, too, preferred stocks play an important role in risk reduction with a strong shift of the frontier to the left when preferred stocks are included (blue curve) relative to the benchmark case (green). Second, inclusion of REITs is important when value indices are not included. Inclusion of REITs only (no preferred stock,) causes an important expansion of the frontier (red) relative to the benchmark case (green). There are both risk reduction effect (shift to the left in the minimum variance portfolio) and improvement in the risk-return trade-off (expansion of the frontier).

Figure 6 illustrates this case for constrained portfolio optimization. Panel A displays the case when Russell 2000 Value and Growth indices are included, but Russell 2000 is not. Panel B illustrates the case when Russell 2000 is included but Russell 2000 Value and Growth indices are both excluded. The conclusions are similar to the unconstrained case, but the effects are stronger. Inclusion of preferred stock yields significant risk reduction relative to the benchmark case--the minimum variance portfolio exhibits a strong decrease in variance.

Table 3, Panels 10 and 11 provides additional evidence. In the unconstrained case, addition of preferred stock and REITs results in a 59.9% Sharpe Ratio improvement when only the Russell 2000 is a part of the set versus a 25.15% improvement when Russell 2000 Value and Growth indices are included (the numbers for the constrained case are 59.8% and 37.26%, respectively.) Inclusion of REITs only, yields a Sharpe Ratio improvement of 25.5% when only the Russell 2000 aggregate index is present versus a 1.8% improvement when both Russell 2000 Value and Growth indices are present (in the constrained case the numbers are 24.62% vs. 7.42%).

Taken together, the three experiments suggest that inclusion of preferred stock and REITs has a larger impact on the optimal portfolios when value indices are not a part of the investment opportunities set. Equity investors who do not have allocations to value and growth stocks as separate asset classes benefit the most from the inclusion of REITs and of preferred stock. These results indicate that REITs offer an important value dimension.

4.2 Debt and Equity Asset Classes

We repeat the previous analysis but now include debt: Investment Grade and High Yield Bond Return indices. The investment opportunity set now includes 13 asset classes: 2 debt and the 11 previously used equity indices. This is our most inclusive investment opportunity set. Data from Table 2 (expected returns, standard deviations and correlations) is used to construct four unconstrained mean-variance frontiers shown in Figure 7: a frontier with debt and all equity asset classes, including preferred stock and REITs (black); a frontier that includes preferred stock, but does not include REITs (blue); a frontier that includes REITs, but does not include preferred stock (red); and a frontier that excludes both preferred stock and REITs (green). The figure illustrates that inclusion of preferred stock or REITs plays a substantially smaller role when bonds are a part of the investors' opportunity set. All four unconstrained frontiers virtually coincide suggesting that in the presence of bond indices, preferred stock no longer play the prominent role they did when bonds were not a part of the investment opportunity set. This suggests that from the point of view of investors, preferred stocks play the role of debt: When debt is not a part of the portfolios, preferred stock contributes significantly to the improvement of the risk-return trade-off, but the contribution of preferred stock is much smaller when fixed income securities are already a part of the investor's portfolio. The intuition of Graham and Dodd who in their 1934 book proposed to place preferred stocks with bonds is confirmed in the Modern Portfolio Theory analysis.

Figure 8 shows efficient frontiers (with and without preferred stock and REITs) when no short sales are allowed, the constrained case. The green curve shows the frontier when preferred stock and REITs are excluded from the investment opportunities set (bonds are included in all four frontiers). Inclusion of preferred stocks and REITs improves risk-return characteristics of the frontier, but in a different way. When preferred stocks are included (but REITs are not), the constrained frontier shifts to the left and portfolio variance decreases for a wide range of expected returns (blue curve). For a wide range of lower variances (and expected returns) the frontier with preferred stock only, overlaps with the frontier that includes both preferred stock and REITs. That is, for investors with relatively high risk aversion, inclusion of preferred stock drives the improvement in the risk-return tradeoff. The effect of including REITs is different. Inclusion of REITs makes portfolios with high expected returns (and high risk) attainable. Investors with relatively low risk aversion seeking portfolios with higher risk (and return) will find the inclusion of REITs to be crucial for the long-only portfolios.

The characteristics of the market (tangency) portfolios are reported in Table 4. Portfolios are formed with the same mixes of equity asset classes as in Table 3, but all cases include two bond asset classes (investment grade and high yield). Both the unconstrained and constrained cases are shown. Panel A corresponds to Figures 7 and 8.

In the unconstrained case, the market portfolio constructed without preferred stock or REITs has an expected return of 0.894% per month, monthly standard deviation of 2.22% and a Sharpe Ratio of 0.2875 (the corresponding annualized Sharpe Ratio is 0.996, a high number relative to what is historically attainable.) When preferred stocks and REITs are added, the Sharpe Ratio improves somewhat, but not by a large

amount. In the constrained case, the market portfolio constructed without preferred or REITs has an expected return of 0.65% per month, monthly standard deviation of 1.718% and a Sharpe Ratio of 0.2310. When preferred stocks and REITs are added, the Sharpe Ratio improves by approximately 2%.[15] While the Sharpe ratio is one metric of diversification benefits, for the constrained case another observation is important. Preferred stocks move the whole frontier to the left. For risk averse investors (who will choose portfolios on the frontier toward the tangency point, but above it) utility is improved because the portfolios with lower risk (for the same return) are now available to them, whereas without preferred stock such portfolios are not attainable. For low risk aversion investors, who desire high rates of return (and are willing to accept higher variances), addition of REITs makes portfolios available that are not attainable in the benchmark case. Therefore, the diversification benefits from preferred stock and REITs accrue to both high and low risk aversion categories in a stronger way than Sharpe Ratio improvements would imply.

4.21 The Value Dimension

Similarly to equity-only investment opportunities sets, we investigate the role that preferred stock and REITs play relative to the value dimension. The experiments are similar. First, we compare contribution of preferred stock and REITs to diversification relative to the inclusion (or exclusion) of Mid-Cap Value and Russell 2000 Value indices. Table 4, Panel 3 reports optimal portfolio characteristics when the aggregate Mid-Cap and Russell 2000 indices are included, but the corresponding value and growth indices are excluded (we exclude Mid-Cap Value, Mid-Cap Growth, Russell 2000 Value and Russell 2000 Growth). The results can be contrasted with Table 4, Panel 4 that describes the case when value and growth indices *are* included (Mid-Cap Value, Mid-Cap Growth, Russell 2000 Value and Russell 2000 Growth), but the aggregate indices (Mid-Cap and Russell 2000) are not. Bonds are included in both cases. When only the aggregate indices are included preferred stock and REITs have a larger impact on the risk-return tradeoff of the optimal portfolios than in the case when the set of investment opportunities already includes Mid-Cap Value and Russell 2000 Value indices. Here we are interested in the *relative* impact of preferred stock and REITs when the value dimension is not a part of the set compared to the case when a value index is already included. The result obtained in both unconstrained and constrained cases: preferred stock and REITs a more important role in the absence of value.

Second, we compare contribution of preferred stock and REITs to diversification relative to inclusion (or exclusion) of Mid-Cap Value and Growth index when all three Russell 2000 indices are excluded. The results are reported in Table 4, Panels 6 and 7. When only the aggregate Mid-Cap index is included, addition of both preferred stock and REITs results in a larger improvement in the Sharpe ratio than in the case when Mid-Cap Value and Growth are a part of the set. This result holds for both the unconstrained and constrained case. The result that REITs play a more important role when the value dimension is not a part of the initial set holds when bonds are a part of the asset set.

Finally, in the third experiment, Mid-Cap, Mid-Cap Value and Growth indices are excluded. Russell 2000 and Russell 2000 Value and Growth indices are used in different combinations. Figure 9 illustrates this experiment for the unconstrained case. When the aggregate Russell 2000 index is excluded but Russell 2000

[15] The magnitude of these Sharpe Ratios is similar to the numbers reported in Goetzmann and Ukhov (2006) study of UK international investment. Using five asset classes (domestic common, domestic preferred, domestic debt, foreign common and foreign debt), they show that in unconstrained case the annualized Sharpe ratio increases from 0.70 in domestic-assets-only case to 0.85 when foreign asset classes are added. In the constrained case, the improvement is from 0.67 to 0.81, or 20.90%.

Value and Growth indices are included (Panel A), the frontiers virtually coincide. However, when Value and Growth indices are not a part of the set, inclusion of preferred stock or REITs results in an important improvement relative to the benchmark case (the benchmark is in green). Inclusion of REITs (or preferred stock) is important when value indices are not included.

Figure 10 illustrates this case for constrained portfolio optimization. Panel A displays the case when Russell 2000 Value and Growth indices are included, but Russell 2000 is not. Panel B illustrates the case when Russell 2000 is included but Russell 2000 Value and Growth indices are both excluded. Similarly to the scenario without bonds, the effects are stronger in the constrained case than in the unconstrained case. Inclusion of preferred stock yields significant risk reduction relative to the benchmark case--the minimum variance portfolio exhibits a strong decrease in variance. Inclusion of REITs is crucial for improvement for portfolios with higher risk and return. The results reported in Table 4, Panels 10 and 11 provide additional support.

Taken together, the three experiments suggest that inclusion of preferred stock and REITs has a larger impact on the optimal portfolios when value indices are not a part of the investment opportunities set, even in the presence of bonds. Even investors who holds bonds, but do not have allocations to value and growth stocks as separate asset classes, will benefit from the inclusion of REITs and of preferred stock. These results indicate that REITs offer an important value dimension even for portfolios that include exposure to fixed income.

4.3 Subsets

The previous analysis can be refined in several ways. While our data includes 13 asset classes in total (2 bonds and 11 equity), investors may limit the number of assets in their portfolios. Barber and Odean (2001) report that the median household held 2.6 stocks in their large individual investor database. When investors face transaction costs, such as brokerage commissions, and have limited funds available, they take transaction costs into account while forming portfolios. Investors may also face costly information processing and limited attention. These factors result in investors not using all assets in the opportunity set, but only investing in a sub-set of assets.

We first assume that investors use seven assets in their portfolios. There are 13 asset classes (11 equity and 2 bond indices.) There are 1716 different subsets of 7 assets drawn from 13. In the unconstrained case, when subsets are ranked by Sharpe Ratio, of the top 20% (343 subsets), 128 subsets include a positive allocation to preferred stock (the average allocation to preferred stock across these 128 subsets is 12.53%). The average Sharpe Ratio for these 128 subsets is 0.27 compared to the average Sharpe Ratio of 0.2716 for the top 343 subsets. Thus preferred stock is an important component of top-performing portfolios. At the same time, 115 of the top 343 subsets contain an allocation to REITs; the average Sharpe Ratio for these 115 subsets equals 0.27. In the 115 cases that include REITs, in 25 cases the weight is positive (average weight is 3.79%) and in 90 cases the weight is negative (average weight is -3.50%), with the overall average weight of -1.91%.

How many top-performing sub-sets include both preferred stocks and REITs? Of the top 343 subsets, only 29 include both preferred stock and REITs. For these 29, the allocation to preferred stock is always positive (average allocation is 13.53%). For 26 out of 29 cases, allocation to REITs is negative (average allocation of -5.0%). For 3 out of 29 cases allocation to REITs is positive (average allocation of 4.27%). In sum, of the top 20% subsets ranked by the Sharpe Ratio (343 subsets out of 1716), only three contain positive allocation to both REITs and preferred stock, with a substantially higher weight assigned to preferred stock.

Now consider the constrained case (no short sales.) When all 1716 different subsets of 7 assets selected from 13 are ranked by Sharpe Ratio, all of the top 20% (343 subsets) include a positive allocation to preferred stock (the average allocation to preferred stock across these 343 subsets is 13.99%). The average Sharpe Ratio for these 343 subsets is 0.2345. At the same time, 147 of the top 343 subsets contain allocation to REITs. The average Sharpe Ratio for these 147 subsets equals 0.2345. Although among the top 343 subsets, 147 contain REITs as a part of the set, the constrained optimization procedure allocates non-zero amount to the REITs only is 7 cases (the average weight among those portfolios is 1.81%). In the remaining cases, the sub-set is such that REITs is a part of it, but the optimal constrained portfolios use other asset classes, and contains zero allocation to REITs.

As a robustness check we also perform the experiment under the assumption that investors use five assets in their portfolios. There are 1287 different subsets of 5 assets selected from 13 (11 equity and two bond indices). In the unconstrained case, when subsets are ranked by Sharpe Ratio, of the top 20% (257 subsets), 65 subsets include a positive allocation to preferred stocks (the average allocation to preferred stocks across these 65 subsets is 16.99%). The average Sharpe ratio for these 65 subsets is 0.2478 compared to the average Sharpe Ratio of 0.2494 for the top 257 subsets. At the same time, 51 of the top 257 subsets include an allocation to REITs. The average Sharpe Ratio for these 51 subsets is 0.2465. In the 51 cases that include REITs, in 21 cases the weight is negative (average weight is -2.35%) and in 30 cases the weight is positive (average weight is 4.15%), with the overall average weight of 1.47%. These results are similar to the ones obtained under the assumption of 7 assets in the sub-sets, supporting the importance of inclusion of the preferred stock.

Of the top 257 subsets, only 6 include both REITs and preferred stock. For these six, the allocation to preferred stock is always positive (average allocation is 14.00%). For 4 out of 6 cases, allocation to REITs is negative (average allocation of -3.28%). For 2 out of 6 cases allocation to REITs is positive (average allocation of 1.66%). In sum, of the top 20% subsets ranked by the Sharpe Ratio (257 subsets out of 1287), only two contain positive allocation to both preferred stock and REITs, with a substantially higher weight assigned to the preferred stock.

Now consider the constrained case. When the 1287 different subsets of 5 assets selected from 13 are ranked by Sharpe Ratio, of the top 20% (257 subsets), 120 subsets include preferred stock (the average allocation to preferred stock across these 120 subsets is 14.62%). The average Sharpe Ratio for these 120 subsets is 0.2337 compared to the average Sharpe Ratio of 0.2321 for the top 257 subsets. At the same time, 88 of the top 257 subsets include REITs. The average Sharpe Ratio for these 88 subsets is 0.2315. In the 88 cases that include REITs, in 71 cases the weight is non-zero (average weight is 2.95%). In the remaining 17 cases, although the set of 5 assets contains REITs, the constrained optimization assigns zero weight to it.

Of the top 257 subsets, only 35 include both preferred stock and REITs. For these 35, the average allocation to preferred stock is 14.71%. For these 35 cases, in 18 cases allocation to REITs is positive (non-zero), with average allocation of 3.65%. In sum, of the top 20% subsets ranked by the Sharpe Ratio (257 subsets out of 1287), only 18 contain positive allocation to both preferred stock and REITs, with a substantially higher weight assigned to the preferred stock.

The analysis of top-performing subsets illustrates the importance of inclusion of REITs and preferred stock. Among portfolios with the highest Sharpe Ratios most include preferred, with allocation to this asset class on the order of 14%.

5 Conclusion

Preferred stock represents a significant portion of the capital structure of REITs, although little attention has been paid to its risk-return characteristics for investors. We study diversification benefits of REIT preferred and common stocks. Taking the view of a long run investor, we conduct our analysis using data from the largest time series available, with our data spanning the period November 1992 to November 2012. We examine the optimal mean-variance portfolios of an investor given access to different classes of assets and establish five main new results. First, preferred stock provides significant diversification benefit to an all-equity investor. With unconstraint portfolio formation, access to preferred stock increases the Sharpe Ratio by between 7.92% and 63.39% and increases the Sharpe Ratio by between 61.82% and 25.03% in constrained portfolios without short sales.

Second, preferred stock appears to behave like a bond substitute: from an investor's perspective preferred stock is debt rather than equity. This provides some evidence that firms should consider them debt when issuing them, because that is largely how they are viewed by investors.

Third, preferred stock provides a venue for risk reduction for constrained investors who have access to bonds. Although they are largely a bond substitute, preferred stock reduces the variance of the optimal portfolio even in the presence of bonds. This suggests that for risk-averse investors, preferred stock will be a valuable addition to their portfolios.

Fourth, for an all equity investor, access to REIT common shares provides an important value dimension. When value indices are not a part of the investment opportunities set, inclusion of REIT common stock results in a substantial improvement in the risk-return tradeoff. Our results indicate that the benefit of REITs in a mixed asset portfolio is very sensitive to the omission or inclusion of value portfolios. This has implications for both academics and investors. For investors, those that already have access to mid-cap and small-cap value stocks are likely to receive much small diversification benefits than those commonly reported. From an academic perspective, it indicates that care should be taken in benchmarking REIT returns. Simply controlling will likely give misleading inferences on performance.

Finally, REIT common shares do provide one distinct benefit. Under the realistic scenario for a long term investor of no short sales, REIT common shares are crucial for achieving portfolios with high expected returns. The inclusion of REIT common shares allows investors to form total return portfolios they would not have been able to otherwise form. The ability to form these portfolios will be of most relevance to low risk aversion investors.

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Figure 1.
Performance of Asset Classes

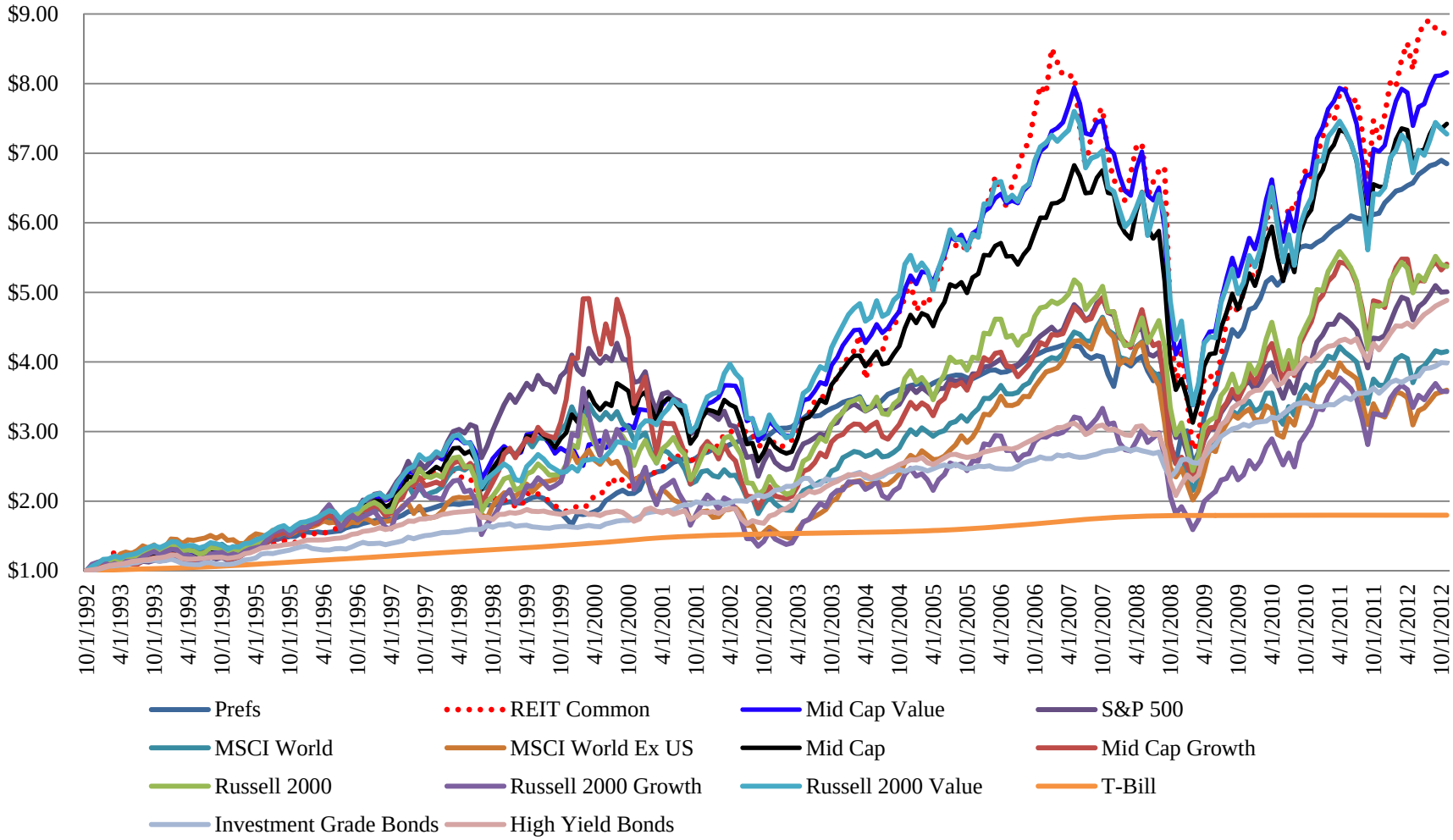


Figure 2.
Performance of Select Equity Asset Classes

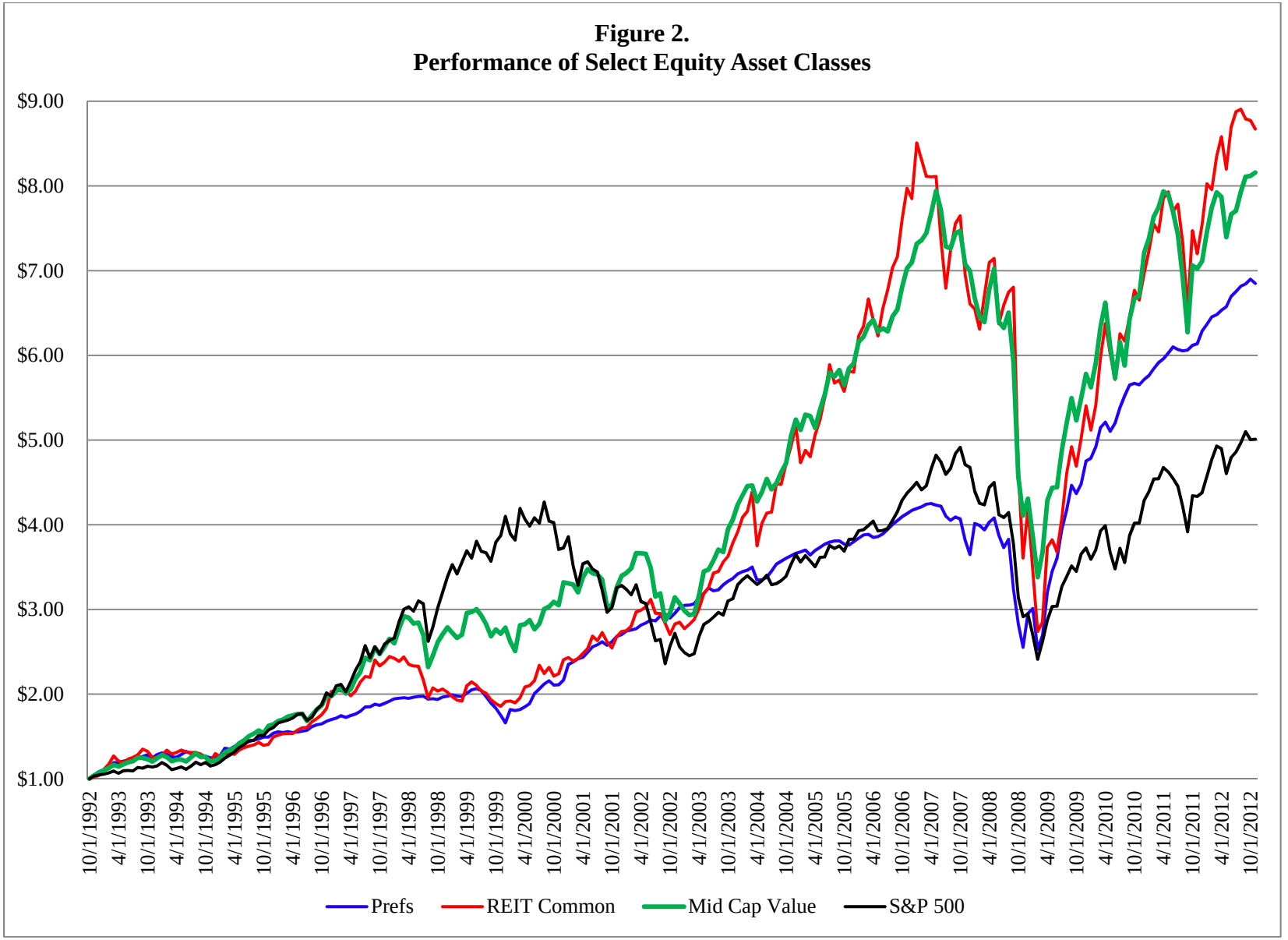


Figure 3: All Equity Asset Classes Unconstrained Efficient Frontiers with and without Preferred Stocks and REITs.

The plot displays four minimum variance frontiers constructed without short-sale restrictions, lines tangent to the frontiers with origin at the risk-free rate, and four tangency portfolios. Equity assets include S&P 500, REIT common, preferred, the world market index, the aggregate mid-cap index and mid-cap value and growth indices, Russell 2000 index, Russell 2000 value and growth, and world ex-US. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs. The data is at monthly frequency.

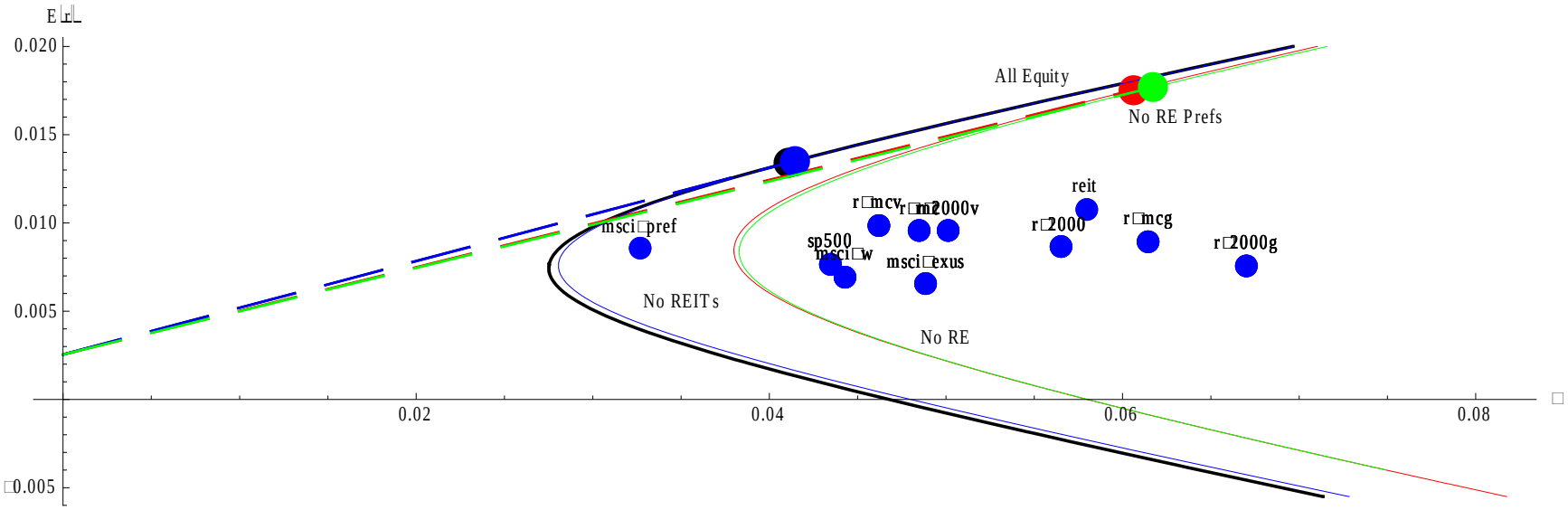


Figure 4: All Equity Asset Classes Constrained Efficient Frontiers with and without Preferred Stocks and REITs.

The plot displays four minimum variance frontiers constructed under short-sale restrictions. Investments in all asset classes are constrained to be positive. Equity assets include S&P 500, REIT common, preferred, the world market index, the aggregate mid-cap index and mid-cap value and growth indices, Russell 2000 index, Russell 2000 value and growth, and world ex-US. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs. The tangency line with origin at the risk-free rate and the tangency (highest Sharpe ratio) portfolio are shown for the portfolio that includes both Preferred and REITs. The data is at monthly frequency.

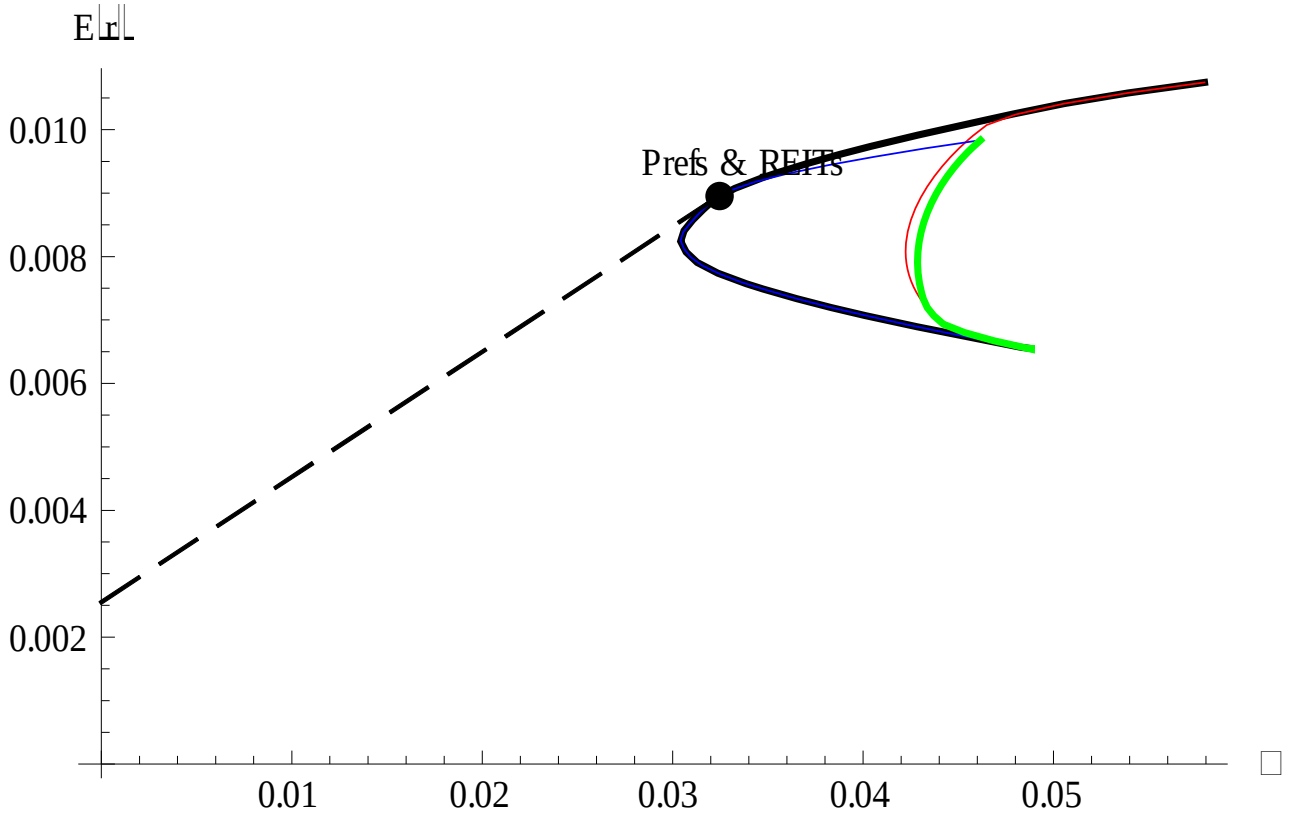
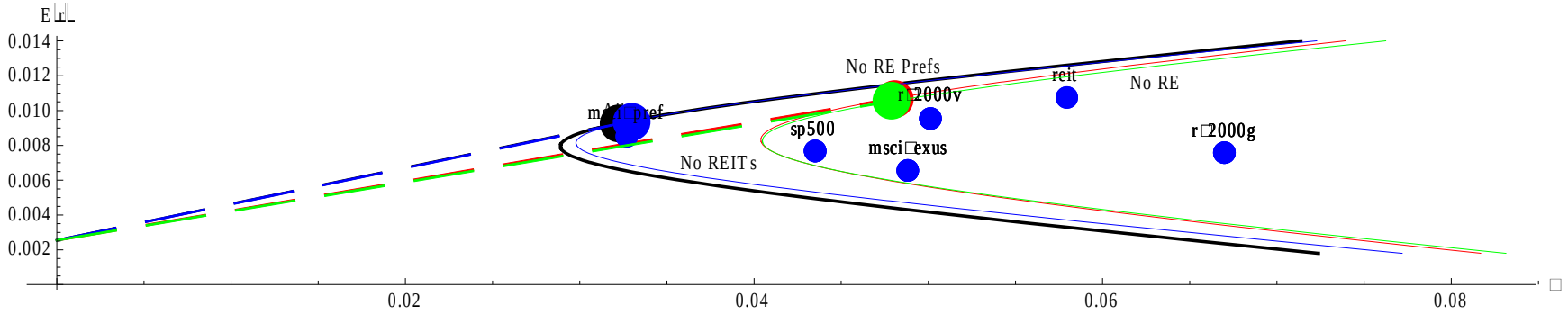


Figure 5: Equity Asset Classes Unconstrained Efficient Frontiers with and without Preferred Stocks and REITs.

The plot displays four minimum variance frontiers constructed without short-sale restrictions, lines tangent to the frontiers with origin at the risk-free rate, and four tangency portfolios. Equity assets include S&P 500, REIT common, preferred, the world market index ex-US. Asset classes not included are the aggregate mid-cap index and mid-cap value and growth indices. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs. The data is at monthly frequency.

Panel A Includes Russell 2000 value and growth indices and excludes Russell 2000.



Panel B Includes Russell 2000 and excludes Russell 2000 value and growth indices.

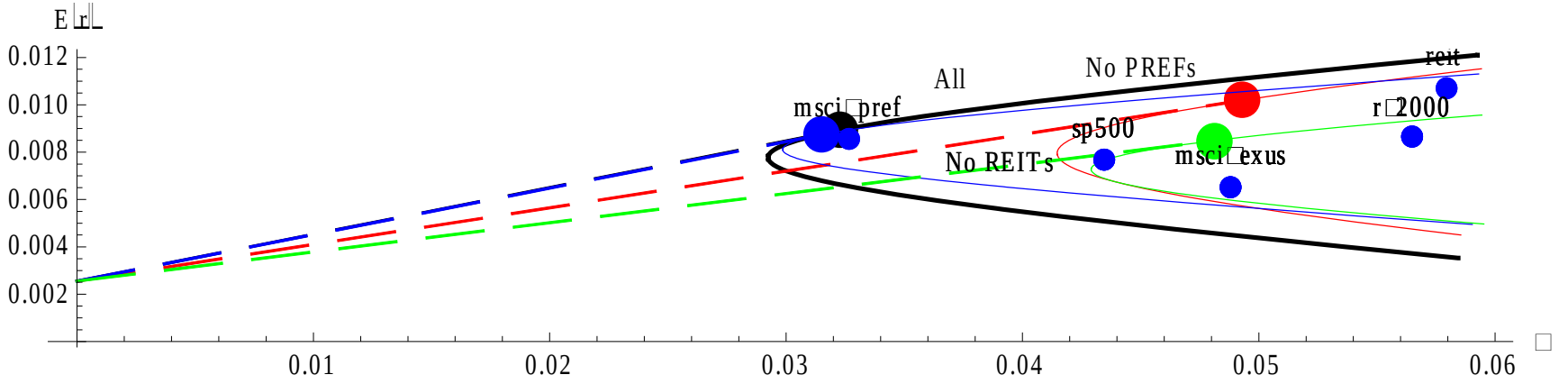


Figure 6: Equity Asset Classes Constrained Efficient Frontiers with and without Preferred Stocks and REITs.

The plot displays four minimum variance frontiers constructed under short-sale restrictions. Investments in all asset classes are constrained to be positive. Equity assets include S&P 500, REIT common, preferred, the world market index ex-US. Asset classes not included are the aggregate mid-cap index and mid-cap value and growth indices. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs. The tangency line with origin at the risk-free rate and the tangency (highest Sharpe ratio) portfolio are shown for the portfolio that includes both Preferred and REITs. The data is at monthly frequency.

Panel A Includes Russell 2000 value and growth indices and excludes Russell 2000.

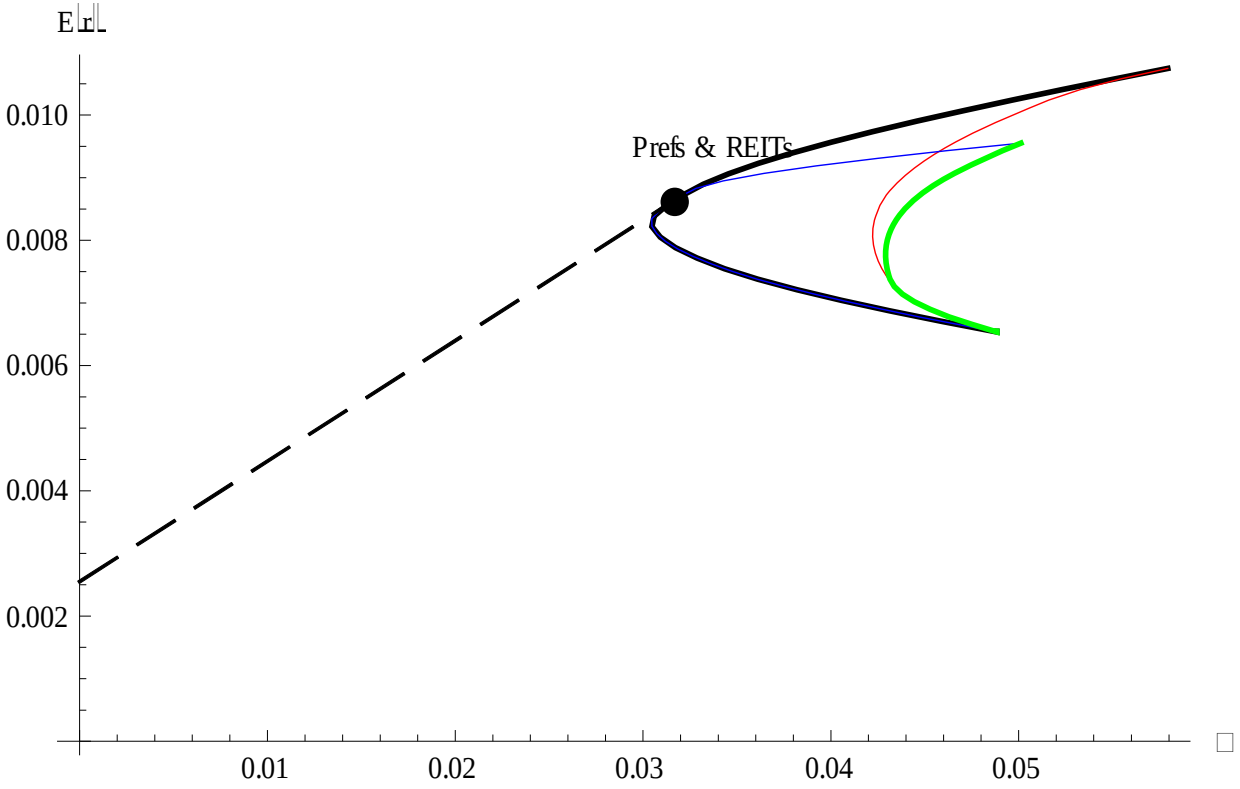


Figure 6 (*Continued*): Equity Asset Classes Constrained Efficient Frontiers with and without Preferred Stock and REITs.

Panel B Includes Russell 2000 and excludes Russell 2000 value and growth indices. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs.

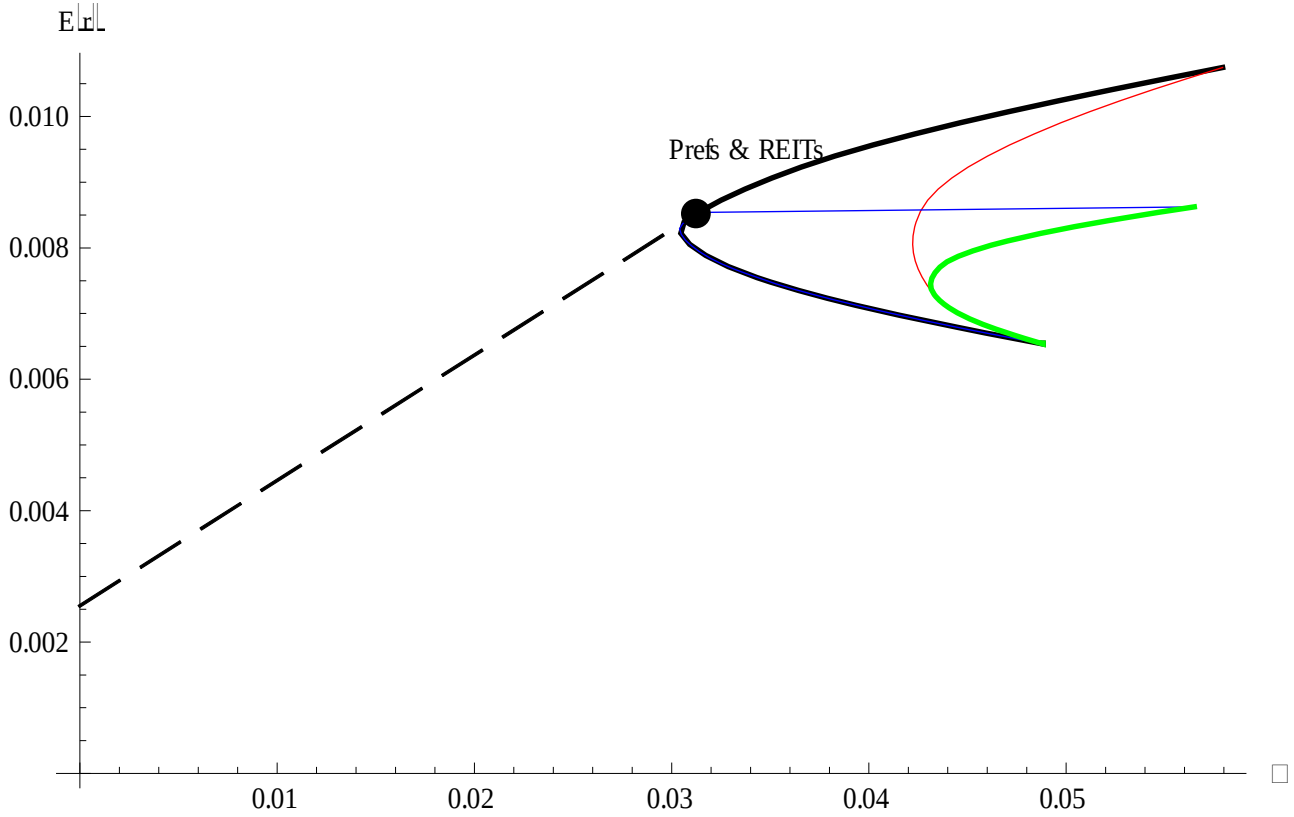


Figure 7: Equity and Bonds Unconstrained Equity Asset Classes (Bonds and Equity) Unconstrained Efficient Frontiers with and without Preferred Stocks and REITs.

The plot displays four minimum variance frontiers constructed without short-sale restrictions, lines tangent to the frontiers with origin at the risk-free rate, and four tangency portfolios. Bond indices are Barclays Investment Grade Debt Index, and Barclays High Yield Debt Index. Equity assets include S&P 500, REIT common, preferred, the world market index, the aggregate mid-cap index and mid-cap value and growth indices, Russell 2000 index, Russell 2000 value and growth, and world ex-US. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs. The data is at monthly frequency.

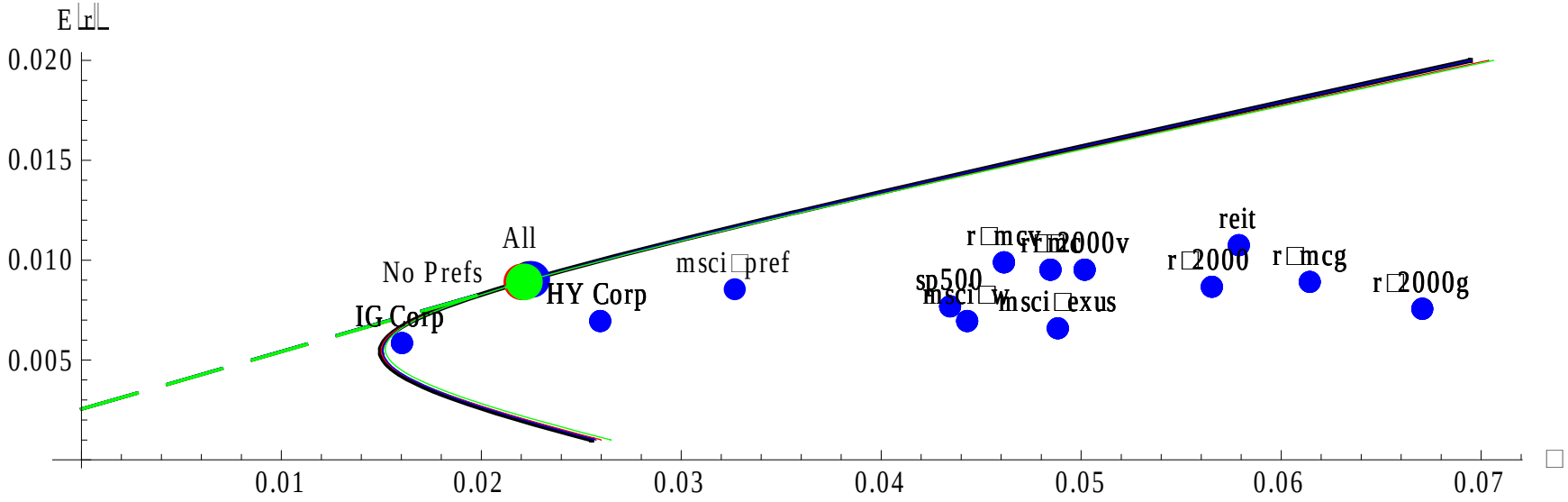


Figure 8: Bonds and Equity Asset Classes Constrained Efficient Frontiers with and without Preferred Stocks and REITs.

The plot displays four minimum variance frontiers constructed under short-sale restrictions. Investments in all asset classes are constrained to be positive. Bond indices are Barclays Investment Grade Debt Index, and Barclays High Yield Debt Index. Equity assets include S&P 500, REIT common, preferred, the world market index, the aggregate mid-cap index and mid-cap value and growth indices, Russell 2000 index, Russell 2000 value and growth, and world ex-US. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs. The tangency line with origin at the risk-free rate and the tangency (highest Sharpe ratio) portfolio are shown for the portfolio that includes both Preferred and REITs. The data is at monthly frequency.

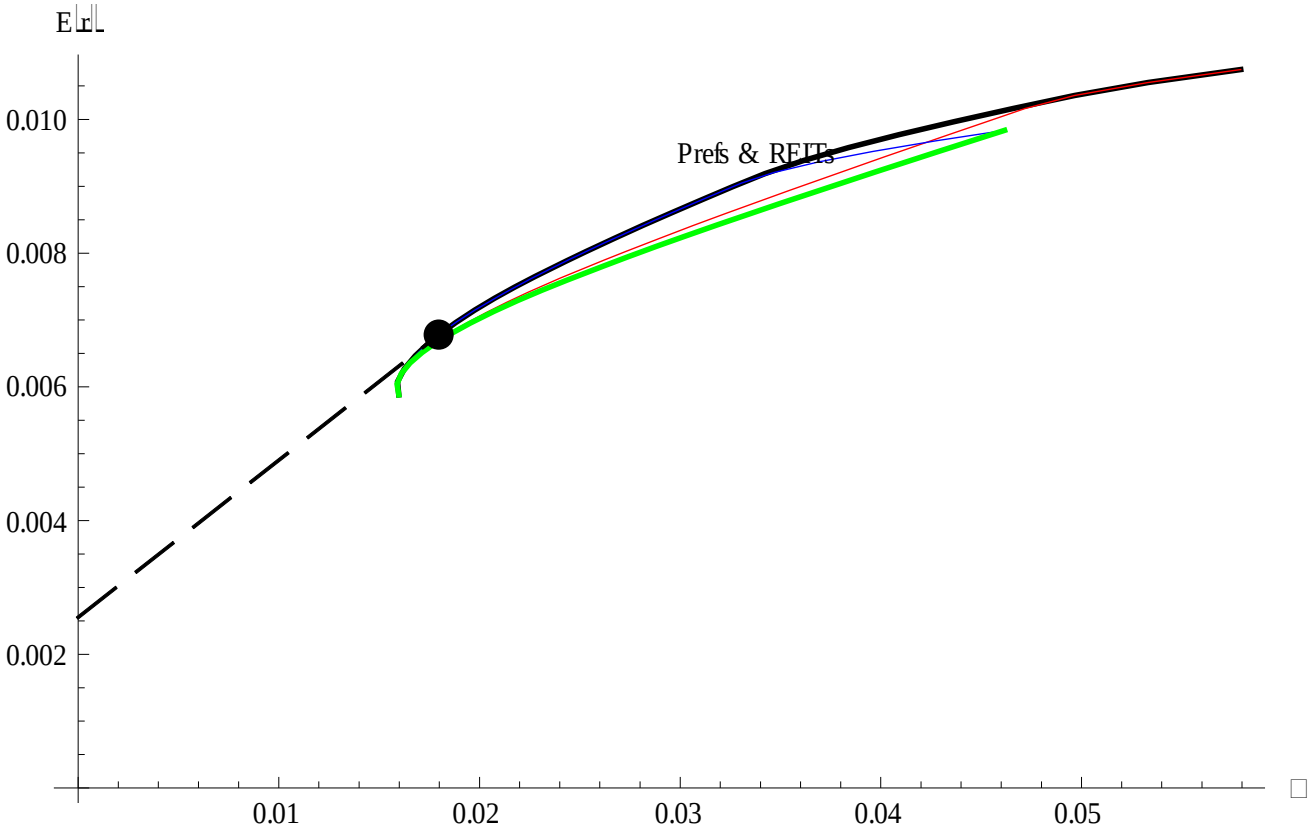
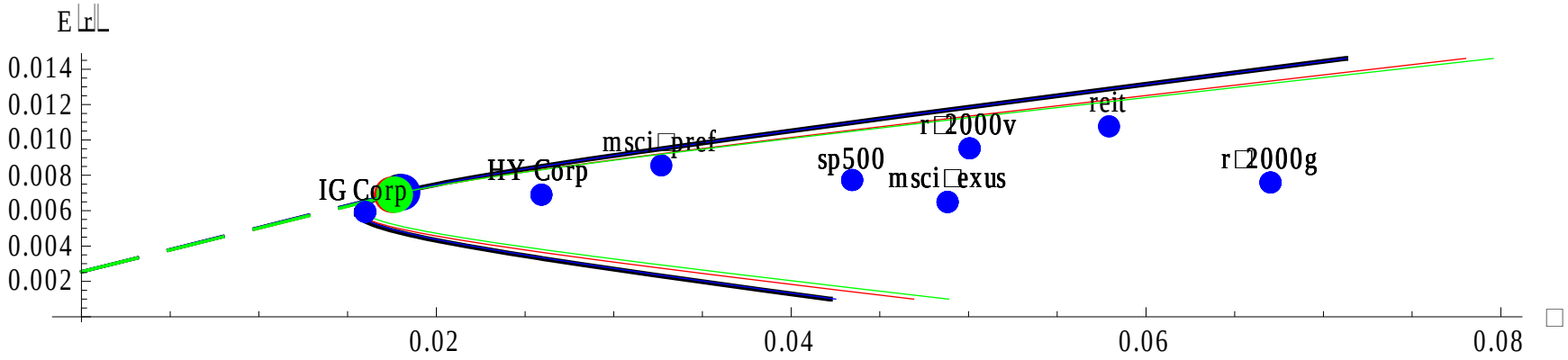


Figure 9: Equity and Bonds Asset Classes Unconstrained Efficient Frontiers with and without Preferred Stocks and REITs.

The plot displays four minimum variance frontiers constructed without short-sale restrictions, lines tangent to the frontiers with origin at the risk-free rate, and four tangency portfolios. Bond indices are Barclays Investment Grade Debt Index, and Barclays High Yield Debt Index. Equity assets include S&P 500, REIT common, preferred, the world market index ex-US. Asset classes not included are the aggregate mid-cap index and mid-cap value and growth indices. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs. The data is at monthly frequency.

Panel A Includes Russell 2000 value and growth indices and excludes Russell 2000.



Panel B Includes Russell 2000 and excludes Russell 2000 value and growth indices.

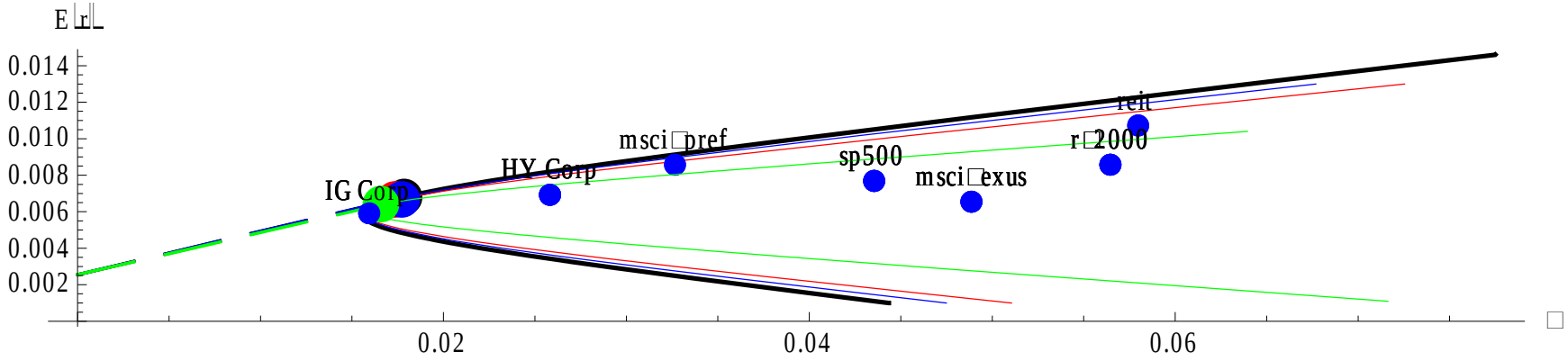


Figure 10: Equity and Bonds Asset Classes Constrained Efficient Frontiers with and without Preferred Stocks and REITs.

The plot displays four minimum variance frontiers constructed under short-sale restrictions. Investments in all asset classes are constrained to be positive. Bond indices are Barclays Investment Grade Debt Index, and Barclays High Yield Debt Index. Equity assets include S&P 500, REIT common, preferred, the world market index ex-US. Asset classes not included are the aggregate mid-cap index and mid-cap value and growth indices. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs. The tangency line with origin at the risk-free rate and the tangency (highest Sharpe ratio) portfolio are shown for the portfolio that includes both Preferred and REITs. The data is at monthly frequency.

Panel A Includes Russell 2000 value and growth indices and excludes Russell 2000.

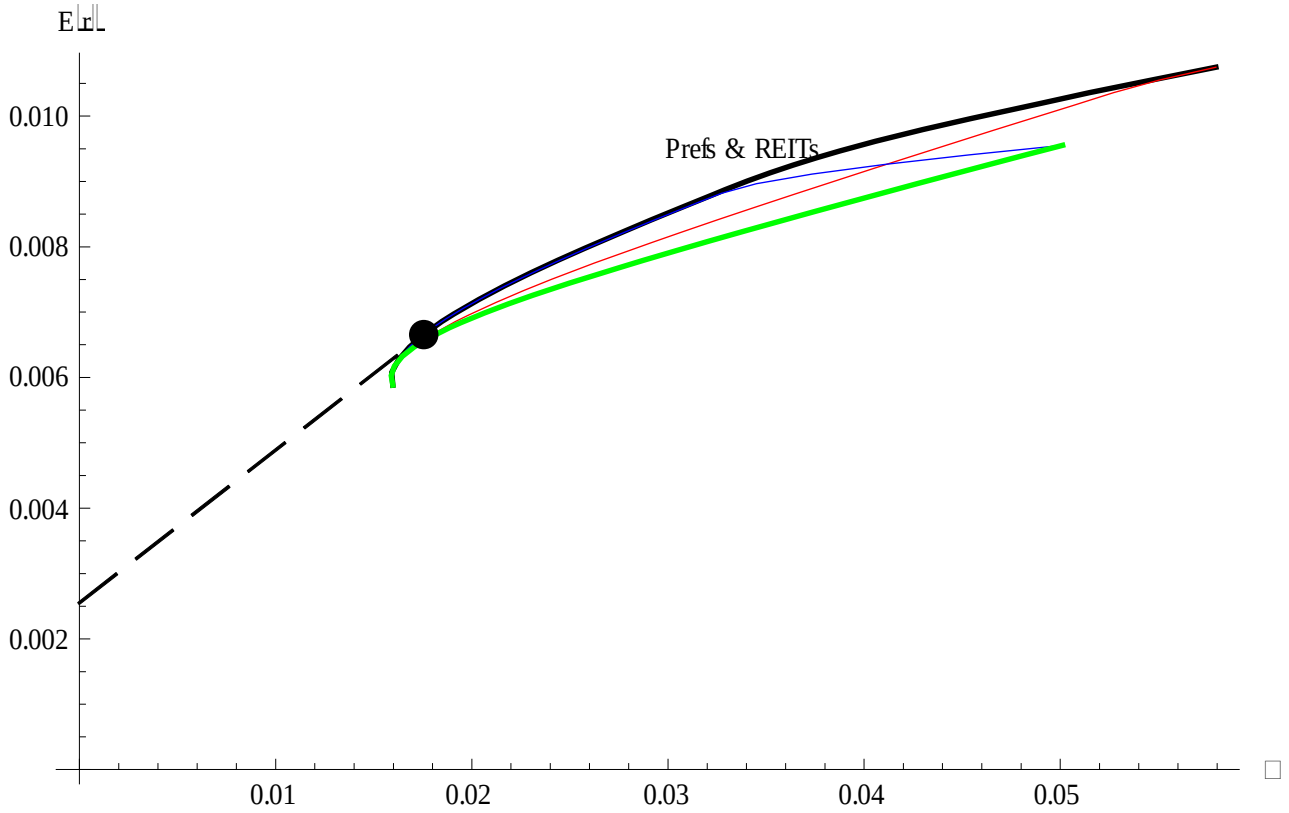


Figure 10 (*Continued*): Equity and Bonds Asset Classes Constrained Efficient Frontiers with and without Preferred Stocks and REITs.

Panel B Includes Russell 2000 and excludes Russell 2000 value and growth indices. The black frontier includes both Preferred and REITs; blue frontier includes Preferred but excludes REITs; red frontier includes REITs but excludes preferred; green frontier excludes both Preferred and REITs.

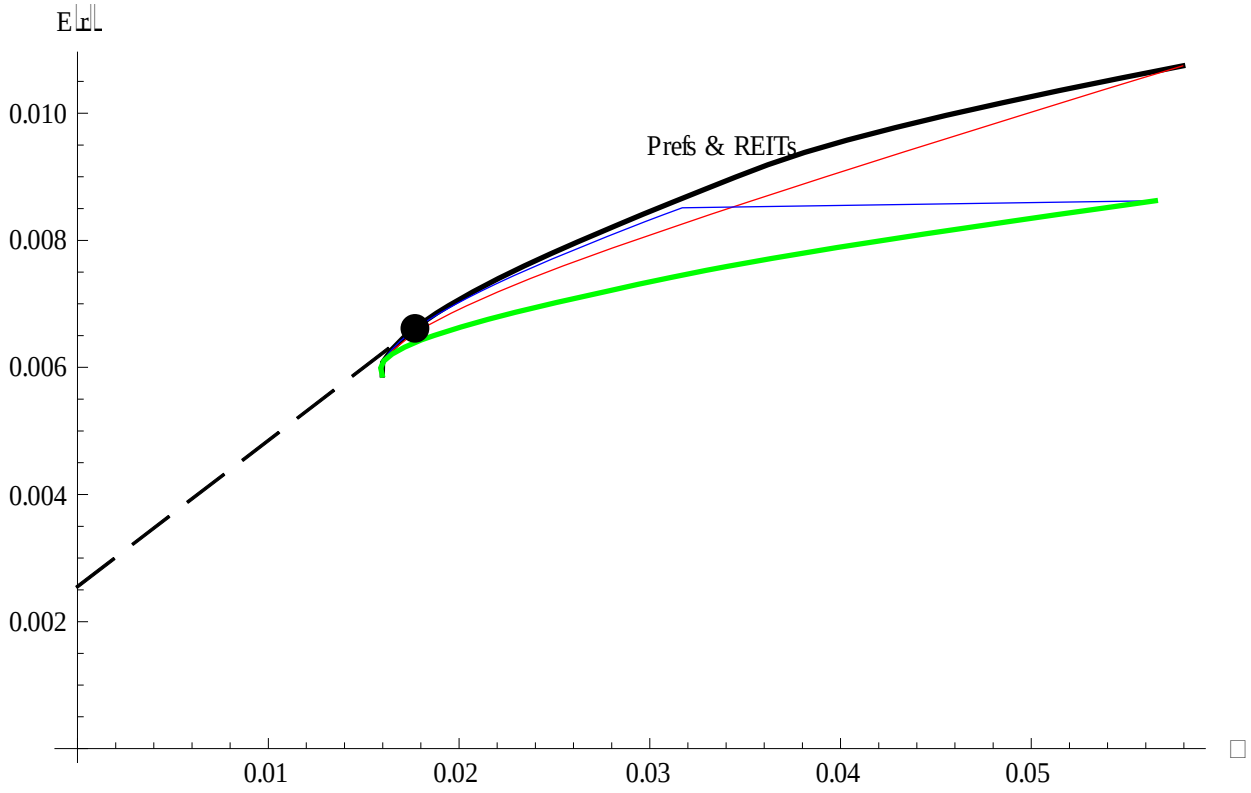


Table 1: Security Issuances

Table reports dollar values of public securities issued by equity REITs from 1992 to 2012. Debt is public debt issuances, IPO is initial public offering of equity, Preferred are public preferred stock issuances, and Seasoned are public seasoned equity offerings. All figures are reported in millions of dollars. Issuance data was obtained from NAREIT (http://www.reit.com/DataAndResearch/REIT-Capital-Offerings/Detailed-Data.aspx).

Year	Debt	IPO	Preferred	Seasoned
1992	310	693	46	808
1993	2,348	8,485	666	2,609
1994	3,173	6,714	155	3,337
1995	3,324	827	1,678	4,727
1996	4,327	1,108	1,550	8,561
1997	9,785	4,776	4,795	19,381
1998	13,941	1,269	4,879	12,006
1999	9,555	292	2,150	1,966
2000	6,045	-	365	1,171
2001	8,650	-	679	1,769
2002	8,353	517	1,067	3,342
2003	9,958	2,325	4,905	4,484
2004	16,956	4,581	4,822	6,698
2005	15,515	1,726	2,735	6,805
2006	24,322	1,824	3,751	13,554
2007	15,765	737	2,998	7,243
2008	4,343	-	947	7,492
2009	10,193	633	-	18,172
2010	18,444	1,987	1,631	18,742
2011	13,525	1,766	4,108	14,782
2012	19,400	1,049	8,466	23,815
Total	218,229	41,309	52,390	181,463
% of Total	44%	8%	11%	37%

Table 2: Returns and Correlations

Table reports descriptive statistics of monthly returns on 13 indices between November 1992 and November 2012. Panel A reports means and standard deviations, while Panel B reports correlations. Pref is the MSCI REIT Preferred index, IG Corp is the Barclays Investment Grade Corporate Bond index, HY Corp is the Barclays High Yield Corporate Bond index, World is the MSCI World index, World Ex-US is the MSCI World Ex-US index, REIT is the SNL Equity REIT index, MidCap is the Russell MidCap index, MidCap Growth is the Russell MidCap Growth index, MidCap Value is the Russell MidCap Value index, Russ2000 is the Russell 2000 index, Rus2000 Growth is the Russell 2000 Growth index, Rus2000 Value is the Russell 2000 value index, and SP500 us the S&P 500 index.

Panel A													
	Pref	IG Corp	HY Corp	World	World Ex-US	REIT	MidCap	MidCap Growth	MidCap Value	Rus2000	Rus2000 Growth	Rus2000 Value	SP500
Monthly													
Mean	0.009	0.006	0.007	0.007	0.007	0.011	0.010	0.009	0.010	0.009	0.008	0.010	0.008
Std	0.033	0.016	0.026	0.044	0.049	0.058	0.049	0.062	0.046	0.057	0.067	0.050	0.044
Annualized													
Mean	0.103	0.071	0.083	0.083	0.078	0.129	0.115	0.107	0.118	0.103	0.091	0.115	0.092
Std	0.114	0.055	0.090	0.154	0.170	0.201	0.168	0.213	0.160	0.196	0.233	0.174	0.151
Panel B: Correlations													
Pref	1.00	0.54	0.68	0.41	0.40	0.62	0.47	0.34	0.52	0.42	0.34	0.49	0.38
IG Corp	0.54	1.00	0.54	0.28	0.29	0.31	0.28	0.21	0.31	0.19	0.16	0.22	0.27
HY Corp	0.68	0.54	1.00	0.65	0.63	0.61	0.68	0.60	0.66	0.63	0.59	0.63	0.62
World	0.41	0.28	0.65	1.00	0.96	0.57	0.90	0.83	0.86	0.81	0.78	0.78	0.94
World Ex-US	0.40	0.29	0.63	0.96	1.00	0.54	0.82	0.75	0.77	0.75	0.72	0.72	0.82
REIT	0.62	0.31	0.61	0.57	0.54	1.00	0.65	0.47	0.75	0.65	0.52	0.77	0.56
MidCap	0.47	0.28	0.68	0.90	0.82	0.65	1.00	0.93	0.93	0.93	0.89	0.90	0.93
MidCap Growth	0.34	0.21	0.60	0.83	0.75	0.47	0.93	1.00	0.74	0.90	0.95	0.76	0.85
MidCap Value	0.52	0.31	0.66	0.86	0.77	0.75	0.93	0.74	1.00	0.82	0.72	0.91	0.88
Rus2000	0.42	0.19	0.63	0.81	0.75	0.65	0.93	0.90	0.82	1.00	0.97	0.94	0.81
Rus2000 Growth	0.34	0.16	0.59	0.78	0.72	0.52	0.89	0.95	0.72	0.97	1.00	0.83	0.78
Rus2000 Value	0.49	0.22	0.63	0.78	0.72	0.77	0.90	0.76	0.91	0.94	0.83	1.00	0.79

Table 3: Optimal Portfolios: Equity Asset Classes

Optimal portfolios are constructed using different investment opportunity sets. For each opportunity set four cases are considered: (a) Both preferred index and REIT Common index are included; (b) REIT Common index is included, but preferred index is not; (c) Preferred index is included, but REIT Common is not; (d) Neither preferred index not REIT Common are included (no real estate). Short sales are allowed in the unconstrained case. No short sales are allowed in the constrained case. The table reports expected return, standard deviation, the Sharpe ratio of the market (tangency) portfolio, and the percent improvement in the Sharpe ratio relative to the benchmark case (d) when no real estate is included. The data is monthly for the period November 1992 through November 2012. The average monthly risk-free rate (1 month T-Bill) is 0.00255. Equity indices are (1) MSCI REIT Preferred Index; (2) MSCI World Index; (3) MSCI World Ex-US Index; (4) SNL REIT Index; (5) S&P 500 Index; (6) Russell Mid Cap Index; (7) Russell Mid Cap Growth Index; (8) Russell Mid Cap Value Index; (9) Russell 2000; (10) Russell 2000 Growth; (11) Russell 2000 Value.

	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)
	Unconstrained				Constrained			
Panel 1: All Equity								
Including Prefs & REITS	0.01342	0.04106	0.2646	7.96%	0.00894	0.03237	0.1973	25.03%
Including REITs, No Prefs	0.01754	0.06061	0.2472	0.86%	0.01008	0.04652	0.1617	2.47%
Including Prefs, No REITs	0.01351	0.04143	0.2645	7.92%	0.00894	0.03237	0.1973	25.03%
No REITs, No Prefs	0.01767	0.06168	0.2451	Base	0.00984	0.04616	0.1578	Base
Panel 2: MSCI World Not Included, The rest are included								
Including Prefs & REITS	0.01315	0.04040	0.2622	8.26%	0.00894	0.03237	0.1973	25.03%
Including REITs, No Prefs	0.01715	0.05988	0.2438	0.66%	0.01008	0.04652	0.1617	2.47%
Including Prefs, No REITs	0.01327	0.04093	0.2619	8.13%	0.00894	0.03237	0.1973	25.03%
No REITs, No Prefs	0.01729	0.06085	0.2422	Base	0.00984	0.04616	0.1578	Base
Panel 3: S&P 500, World Ex-US, MidCap, Russell 2000 (No MSCI World, No Midcap V/G, No Russell 2000 V/G)								
Including Prefs & REITS	0.01179	0.04158	0.2223	19.58%	0.00881	0.03205	0.1951	35.20%
Including REITs, No Prefs	0.01651	0.07029	0.1986	6.83%	0.01007	0.04782	0.1571	8.87%
Including Prefs, No REITs	0.01162	0.04087	0.2219	19.37%	0.00881	0.03205	0.1951	35.20%
No REITs, No Prefs	0.01714	0.07847	0.1859	Base	0.00955	0.04851	0.1443	Base
Panel 4: S&P 500, World Ex-US, MidCap V/G, Russell 2000 V/G (No MSCI World, No MidCap, No Russell 2000)								
Including Prefs & REITS	0.01255	0.04139	0.242	10.50%	0.00894	0.03237	0.1973	25.03%
Including REITs, No Prefs	0.01720	0.06656	0.220	0.46%	0.01008	0.04652	0.1617	2.47%
Including Prefs, No REITs	0.01275	0.04235	0.241	10.05%	0.00894	0.03237	0.1973	25.03%
No REITs, No Prefs	0.01729	0.06728	0.219	Base	0.00984	0.04616	0.1578	Base

	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)
	Unconstrained				Constrained			
Panel 5 (MidCap and no Russell 2000): S&P 500, World Ex-US, MidCap, MidCap V and G (No MSCI World, No Russell 2000, No Russell 2000 V or G)								
Including Prefs & REITS	0.01031	0.03550	0.2180	18.54%	0.00894	0.0324	0.1973	25.03%
Including REITs, No Prefs	0.01305	0.05632	0.1860	1.14%	0.01008	0.04652	0.1617	2.47%
Including Prefs, No REITs	0.01037	0.03587	0.2179	18.49%	0.00894	0.03237	0.1973	25.03%
No REITs, No Prefs	0.01324	0.05807	0.1839	Base	0.00984	0.04616	0.1578	Base
Panel 6 (Mid Cap and no Russell 2000): S&P 500, World Ex-US, MidCap V and G (No MSCI World, No Russell 2000, No Russell 2000 V or G, No MidCap)								
Including Prefs & REITS	0.01018	0.03652	0.2088	20.76%	0.00894	0.03237	0.1973	25.03%
Including REITs, No Prefs	0.01322	0.06099	0.1748	1.10%	0.01008	0.04652	0.1617	2.47%
Including Prefs, No REITs	0.01026	0.03706	0.2079	20.24%	0.00894	0.03237	0.1973	25.03%
No REITs, No Prefs	0.01341	0.06277	0.1729	Base	0.00984	0.04616	0.1578	Base
Panel 7 (MidCap and no Russell 2000): S&P 500, World Ex-US, MidCap (No MSCI World, No Russell 2000, No Russell 2000 V or G, No MidCap V or G)								
Including Prefs & REITS	0.01013	0.03666	0.2067	29.27%	0.00881	0.03205	0.1951	35.20%
Including REITs, No Prefs	0.01325	0.06259	0.1708	6.82%	0.01007	0.04782	0.1571	8.87%
Including Prefs, No REITs	0.01014	0.03670	0.2067	29.27%	0.00881	0.03205	0.1951	35.20%
No REITs, No Prefs	0.01348	0.06833	0.1599	Base	0.00955	0.04851	0.1443	Base
Panel 8 (Most Restricted Set): S&P 500, World Ex-US (Not Included: MSCI World, MidCap, MidCap V or G, Russell 2000, Russell 2000 V or G)								
Including Prefs & REITS	0.00894	0.03238	0.1973	64.14%	0.00851	0.03121	0.1908	62.24%
Including REITs, No Prefs	0.01026	0.04987	0.1546	28.62%	0.00950	0.04670	0.1488	26.53%
Including Prefs, No REITs	0.00872	0.03138	0.1964	63.39%	0.00837	0.03057	0.1903	61.82%
No REITs, No Prefs	0.00804	0.04568	0.1202	Base	0.00767	0.04349	0.1176	Base

	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)
	Unconstrained				Constrained			
Panel 9 (Russell 2000 and no MidCap): S&P 500, World Ex-US, Russell 2000, Russell 2000 V and G (No MSCI World, No MidCap, No MidCap V or G)								
Including Prefs & REITS	0.00977	0.03193	0.2261	21.69%	0.00863	0.03162	0.1923	37.26%
Including REITs, No Prefs	0.01138	0.04694	0.1881	1.24%	0.00969	0.04744	0.1505	7.42%
Including Prefs, No REITs	0.0991	0.03273	0.2247	20.94%	0.00863	0.03162	0.1923	37.26%
No REITs, No Prefs	0.01126	0.04686	0.1858	Base	0.00926	0.04787	0.1401	Base
Panel 10 (Russell 2000 and no Mid Cap): S&P 500, World Ex-US, Russell 2000 V and G (No MSCI World, No Russell 2000, No MidCap, No MidCap V or G)								
Including Prefs & REITS	0.00928	0.03225	0.2085	25.15%	0.00863	0.03162	0.1923	37.26%
Including REITs, No Prefs	0.01070	0.04803	0.1696	1.80%	0.00969	0.04744	0.1505	7.42%
Including Prefs, No REITs	0.00940	0.03297	0.2075	24.55%	0.00863	0.03162	0.1923	37.25%
No REITs, No Prefs	0.01053	0.04787	0.1666	Base	0.00926	0.04787	0.1401	Base
Panel 11 (Russell 2000 and no Mid Cap): S&P 500, World Ex-US, Russell 2000 (No MSCI World, No Russell 2000 V or G, No MidCap, No MidCap V or G)								
Including Prefs & REITS	0.00893	0.03231	0.1973	59.89%	0.00851	0.03121	0.1908	59.80%
Including REITs, No Prefs	0.01019	0.04929	0.1549	25.53%	0.00950	0.04670	0.1488	24.62%
Including Prefs, No REITs	0.00873	0.03147	0.1964	59.16%	0.00837	0.03057	0.1903	59.38%
No REITs, No Prefs	0.00849	0.04811	0.1234	Base	0.00789	0.04470	0.1194	Base

Table 4: Optimal Portfolios: Debt and Equity Asset Classes

Optimal portfolios are constructed using different investment opportunity sets. For each opportunity set four cases are considered: (a) Both preferred index and REIT Common index are included; (b) REIT Common index is included, but preferred index is not; (c) Preferred index is included, but REIT Common is not; (d) Neither preferred index not REIT Common are included (no real estate). Short sales are allowed in the unconstrained case. No short sales are allowed in the constrained case. The table reports expected return, standard deviation, the Sharpe ratio of the market (tangency) portfolio, and the percent improvement in the Sharpe ratio relative to the benchmark case (d) when no real estate is included. The data is monthly for the period November 1992 through November 2012. The average monthly risk-free rate (1 month T-Bill) is 0.00255. Bond indices are: (1) Barclays Investment Grade Debt Index; (2) Barclays High Yield Debt Index. Equity indices are (1) MSCI REIT Preferred Index; (2) MSCI World Index; (3) MSCI World Ex-US Index; (4) SNL REIT Index; (5) S&P 500 Index; (6) Russell Mid Cap Index; (7) Russell Mid Cap Growth Index; (8) Russell Mid Cap Value Index; (9) Russell 2000; (10) Russell 2000 Growth; (11) Russell 2000 Value.

	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)
	Unconstrained				Constrained			
Panel 1: All Equity								
Including Prefs & REITS	0.00901	0.02236	0.2886	0.38%	0.00676	0.01790	0.2350	1.73%
Including REITs, No Prefs	0.00891	0.02208	0.2876	0.03%	0.00656	0.01731	0.2312	0.09%
Including Prefs, No REITs	0.00906	0.02255	0.2883	0.28%	0.00676	0.01790	0.2350	1.73%
No REITs, No Prefs	0.00894	0.02221	0.2875	Base	0.00652	0.01718	0.2310	Base
Panel 2: MSCI World Not Included, The rest are included								
Including Prefs & REITS	0.00890	0.02203	0.2881	0.38%	0.00676	0.01790	0.2350	1.73%
Including REITs, No Prefs	0.00880	0.02176	0.2872	0.07%	0.00656	0.01731	0.2312	0.09%
Including Prefs, No REITs	0.00894	0.02221	0.2877	0.24%	0.00676	0.01790	0.2350	1.73%
No REITs, No Prefs	0.00884	0.02189	0.2870	Base	0.00652	0.01718	0.2310	Base
Panel 3: S&P 500, World Ex-US, MidCap, Russell 2000 (No MSCI World, No Midcap V/G, No Russell 2000 V/G)								
Including Prefs & REITS	0.00789	0.02096	0.2546	1.56%	0.00674	0.01794	0.2332	2.73%
Including REITs, No Prefs	0.00776	0.02060	0.2528	0.84%	0.00654	0.01739	0.2291	0.93%
Including Prefs, No REITs	0.00783	0.02077	0.2540	1.32%	0.00672	0.01789	0.2330	2.64%
No REITs, No Prefs	0.00758	0.02007	0.2507	Base	0.00641	0.01697	0.2270	Base
Panel 4: S&P 500, World Ex-US, MidCap V/G, Russell 2000 V/G (No MSCI World, No MidCap, No Russell 2000)								
Including Prefs & REITS	0.00831	0.02080	0.2766	0.44%	0.00676	0.01790	0.2350	1.73%
Including REITs, No Prefs	0.00824	0.02059	0.2759	0.18%	0.00656	0.01731	0.2312	0.09%
Including Prefs, No REITs	0.00835	0.02103	0.2757	0.11%	0.00676	0.01790	0.2350	1.73%
No REITs, No Prefs	0.00829	0.02083	0.2754	Base	0.00652	0.01718	0.2310	Base

	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)
	Unconstrained				Constrained			
Panel 5 (MidCap and no Russell 2000): S&P 500, World Ex-US, MidCap, MidCap V and G (No MSCI World, No Russell 2000, No Russell 2000 V or G)								
Including Prefs & REITS	0.00752	0.01956	0.2539	1.07%	0.00678	0.01801	0.2349	1.86%
Including REITs, No Prefs	0.00735	0.01908	0.2513	0.04%	0.00658	0.01745	0.2310	0.17%
Including Prefs, No REITs	0.00752	0.01958	0.2538	1.04%	0.00678	0.01801	0.2349	1.86%
No REITs, No Prefs	0.00734	0.01906	0.2512	Base	0.00654	0.01730	0.2306	Base
Panel 6 (Mid Cap and no Russell 2000): S&P 500, World Ex-US, MidCap V and G (No MSCI World, No Russell 2000, No Russell 2000 V or G, No MidCap)								
Including Prefs & REITS	0.00734	0.01924	0.2489	0.93%	0.00678	0.01801	0.2349	1.86%
Including REITs, No Prefs	0.00719	0.01881	0.2466	0.00%	0.00658	0.01745	0.2310	0.17%
Including Prefs, No REITs	0.00734	0.01926	0.2486	0.81%	0.00678	0.01801	0.2349	1.86%
No REITs, No Prefs	0.00719	0.01882	0.2466	Base	0.00654	0.01730	0.2306	Base
Panel 7 (MidCap and no Russell 2000): S&P 500, World Ex-US, MidCap (No MSCI World, No Russell 2000, No Russell 2000 V or G, No MidCap V or G)								
Including Prefs & REITS	0.00731	0.01922	0.2474	1.64%	0.00674	0.01794	0.2332	2.73%
Including REITs, No Prefs	0.00713	0.01872	0.2446	0.49%	0.00654	0.01739	0.2291	0.93%
Including Prefs, No REITs	0.00730	0.01918	0.2473	1.60%	0.00672	0.01789	0.2330	2.64%
No REITs, No Prefs	0.00704	0.01844	0.2434	Base	0.00641	0.01697	0.2270	Base
Panel 8 (Most Restricted Set): S&P 500, World Ex-US (Not Included: MSCI World, MidCap, MidCap V or G, Russell 2000, Russell 2000 V or G)								
Including Prefs & REITS	0.00682	0.01794	0.2376	3.85%	0.00663	0.01773	0.2298	3.93%
Including REITs, No Prefs	0.00663	0.01741	0.2343	2.40%	0.00642	0.01713	0.2258	2.13%
Including Prefs, No REITs	0.00671	0.01763	0.2358	3.06%	0.00655	0.01748	0.2285	3.35%
No REITs, No Prefs	0.00634	0.01654	0.2288	Base	0.00617	0.01633	0.2211	Base

	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)
	Unconstrained				Constrained			
Panel 9 (Russell 2000 and no MidCap): S&P 500, World Ex-US, Russell 2000, Russell 2000 V and G (No MSCI World, No MidCap, No MidCap V or G)								
Including Prefs & REITS	0.00738	0.01859	0.2596	1.33%	0.00667	0.01759	0.2338	1.87%
Including REITs, No Prefs	0.00720	0.01808	0.2567	0.20%	0.00646	0.01698	0.2298	0.13%
Including Prefs, No REITs	0.00742	0.01884	0.2582	0.78%	0.00667	0.01759	0.2338	1.87%
No REITs, No Prefs	0.00724	0.01830	0.2562	Base	0.00641	0.01681	0.2295	Base
Panel 10 (Russell 2000 and no Mid Cap): S&P 500, World Ex-US, Russell 2000 V and G (No MSCI World, No Russell 2000, No MidCap, No MidCap V or G)								
Including Prefs & REITS	0.00702	0.01788	0.2500	1.09%	0.00667	0.01759	0.2338	1.87%
Including REITs, No Prefs	0.00688	0.01747	0.2477	0.16%	0.00646	0.01698	0.2298	0.13%
Including Prefs, No REITs	0.00706	0.01809	0.2489	0.65%	0.00667	0.01759	0.2338	1.87%
No REITs, No Prefs	0.00692	0.01765	0.2473	Base	0.00641	0.01681	0.2295	Base
Panel 11 (Russell 2000 and no Mid Cap): S&P 500, World Ex-US, Russell 2000 (No MSCI World, No Russell 2000 V or G, No MidCap, No MidCap V or G)								
Including Prefs & REITS	0.00682	0.01791	0.2381	3.25%	0.00663	0.01771	0.2298	3.61%
Including REITs, No Prefs	0.00663	0.01737	0.2346	1.73%	0.00642	0.01712	0.2258	1.80%
Including Prefs, No REITs	0.00674	0.01766	0.2369	2.73%	0.00655	0.01748	0.2288	3.16%
No REITs, No Prefs	0.00639	0.01663	0.2306	Base	0.00619	0.01638	0.2218	Base