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Room-Risk Management at Sunquest Vacation

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Recommended Citation
Anderson, C. K., & Xie, X. (2009). Room-risk management at Sunquest vacations [Electronic version]. *Cornell Hospitality Quarterly, 50*(3), 314-324. Retrieved [insert date], from Cornell University, School of Hospitality Administration site: http://scholarship.sha.cornell.edu/articles/161/

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Abstract
This article outlines some of the basic complexities that originate with the acquisition of hotel rooms for a reseller of bundled vacations. A tour operator typically acquires or contracts for service capacity, bundles the services (air, hotel, food and beverage, and excursions), then markets and sells to consumers.

The article focuses on the short-term aspects of room-risk management for the tour operator, specifically how to manage blocks of take-or-pay contracted rooms. The room-risk management problem is formulated as a math program with the objective of minimizing wasted rooms. While the exposition focuses on a particular reseller of packaged vacations, the method is applicable to any firm acquiring capacity on take-or-pay contracts and reselling this capacity as bundled vacations.

Keywords
tour operators, revenue management, bundled package sales, Sunquest Tours

Disciplines
Hospitality Administration and Management

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DOI: 10.1177/1938965509336064
Volume 50, Issue 3 314-324

Room-Risk Management at Sunquest Vacations

by CHRIS K. ANDERSON and XIAOQING XIE

This article outlines some of the basic complexities that originate with the acquisition of hotel rooms for a reseller of bundled vacations. A tour operator typically acquires or contracts for service capacity, bundles the services (air, hotel, food and beverage, and excursions), then markets and sells to consumers. The article focuses on the short-term aspects of room-risk management for the tour operator, specifically how to manage blocks of take-or-pay contracted rooms. The room-risk management problem is formulated as a math program with the objective of minimizing wasted rooms. While the exposition focuses on a particular reseller of packaged vacations, the method is applicable to any firm acquiring capacity on take-or-pay contracts and reselling this capacity as bundled vacations.

Keywords: tour operators; revenue management; bundled package sales; Sunquest Tours

Summary

Tour operators face a considerable risk when they contract far in advance for hotel rooms that they will then resell. These "at-risk" rooms are typically sold as part of packages for multiple-day stays (often one week), which means that when customers purchase packages, sufficient rooms must be available for the entire package period. Room availability is also subject to stayovers from packages sold for previous dates. The room-risk challenge can be modeled, with a goal of minimizing unusable rooms under contract. The specific example of Sunquest Vacations, presented here, shows a reduction of 25 percent in spoilage. The tour operator's room availability can be augmented by additional rooms purchased on the spot market. Although the operator is not financially liable for those rooms, they are subject to prior sale and are more expensive to use than the base supply of at-risk rooms. The model is expanded to show the influence of adding the nonrisk rooms to inventory, again with the goal of minimizing spoilage of the contracted, at-risk inventory. This shows an additional reduction of 2 percent in spoilage.

Introduction

Sunquest Vacations, Canada's number one travel provider, is a wholly owned subsidiary of MyTravel Group PLC, which is headquartered in the United Kingdom. Sunquest has been voted by the public as the winner of the Consumer's Choice Award for Excellence four years in a row. More than half a million vacationers travel with Sunquest each year to destinations such as Mexico, the Caribbean, Central America, the United Kingdom, and Ireland, departing from fifteen Canadian locations. In the midst of this good news, Sunquest, like all tour operators, faces a challenging operational environment as it deals with variable supply and demand in a highly competitive landscape. The typical tour operator owns little or no capacity in the final product it sells. Sunquest does own some aircraft capacity that is operated for it under an operating agreement, and it contracts for additional seats with several charter airlines. Some tour operators may choose to contract all their airline capacity, while others may own all their flight capacity.

In addition to providing sufficient airline capacity, the tour operator must also ensure that it has sufficient hotel-room availability, because many of its packages are sold as all-inclusive—covering the flight, room, food and beverage, and airport transfers. Sunquest uses two types of agreements for rooms, one in which the tour operator has full financial risk for the rooms and one in which there is no risk. In a full-risk agreement, the tour operator agrees to block a set number of rooms at a specified price for an agreed-upon arrival period. The tour operator is fully financially responsible for this room block regardless of whatever packages it might sell. It might, for example, contract for a block of one hundred rooms per night for the months of December and January. By contrast, nonrisk rooms are available for purchase by the tour operator at a contracted rate, but the tour operator has no financial obligation for these rooms, with the ability to resell the room (and food and beverage) at some contracted rate. Despite the label we gave them, nonrisk rooms are not without risk entirely, as the tour operator receives no guarantee on availability. The rooms are not blocked, and they are subject to prior sale. The hotelier (probably also revenue managing and closing out deeply discounted rates) can issue a stop sell, removing all room availability for a given arrival period. The random elements in the supply resulting from hotel stop sells is generally not an issue except for a few extremely busy periods (e.g., the week between December 25 and January 2 and spring break periods).

The tour operator's glossy brochure advertising all of its vacation locations and associated hotels—including prices that are effectively rack rates—is published and sent to travel agents well in advance of the vacation season. To stimulate bookings, the tour operator offers discounted prices and promotions. From the consumer's standpoint, prices typically decline with time, but the availability of desirable hotels, room styles, or views likewise diminishes. This situation is similar to that of the cruise industry, where booking prices decline as the exterior cabins with views are reserved, leaving small, interior cabins available for sale.

Tour Operator Revenue Management

Our focus in this article is on the management of full-risk rooms, which falls under Sunquest's revenue-management operation. Tour or bundled vacation packages are a relatively new arena for the direct application of revenue management (RM), and we have seen little published research directed towards RM for tour operators. Talluri and van Ryzin (2004) briefly mention tour as an area of RM application with some popular press discussions of the JDS (formerly Manugistics) RM rollout at Tui in Europe. The absence of published studies does not mean that tour operators are not active practitioners of RM concepts; in fact, they are assertive in the general application of RM fundamentals. Most tour operators have teams of analysts responsible for markets who actively manage prices across a set of distribution channels that continues to expand. However, the industry has been slow to formalize the application of RM tools or systems.

Traditional RM focuses on inventory allocation decisions across demand segments. The early approaches of Littlewood (1972) and Belobaba (1989) were focused on the allocation of capacity across a set of product classes. Williamson and Belobaba (1988) discuss the use of optimization and linear programming for the control of inventory in complex settings like the network of a large airline or a hotel trying to manage rates across a set of different lengths of stay. Gallego and Phillips (2004) provide packaged holidays and bundling as an example of how airlines can create flexible products for the improved management of their supply. Anderson and Marcus (2007) present a summary of revenue management issues for the tour operator and develop a two-period game theoretic model between the tour operator and the hotel operator. Their efforts focus on the optimal mix of risk versus nonrisk rooms that the hotel should offer to the tour operator. Buhalis and Licata (2002) provide an interesting discussion of the migration of tourism distribution to a less consolidated framework. Hoontrankul and Sahadev (2006) provide an excellent introduction to the tour industry with a case study about MoreThailand.com. For a more general discussion of the tour industry, see Cooper et al. (2005). The focus of the model developed in the following sections is on the optimal use of at-risk hotel rooms, which the tour operator has already purchased or contracted for on a take-or-pay basis. Selling prices for the tour operator are competitively set in the marketplace. So instead of addressing price, the model is designed to reduce wasted rooms or, stated differently, to minimize the costs of the hotel rooms sold as part of packaged vacations.

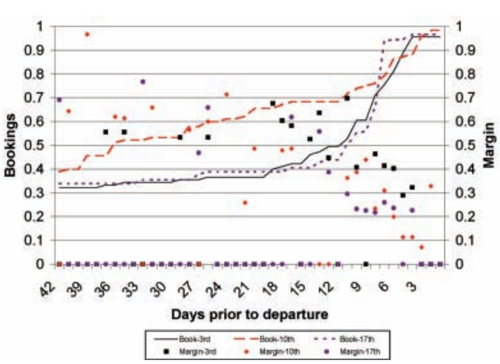
Room Risk at Sunquest Vacations

Exhibit 1 presents a snapshot of seven hotels (labeled A through G for our purposes) during a fourteen-day window in the winter of 2005-2006 for a particular Caribbean location. The table indicates total airline seats available as well as rooms required. The seats-available row indicates how many seats are empty on flights for all departure gateways for a particular arrival location and arrival date. The seats-available number is variable as it aggregates all fifteen Canadian gateways serving the market in question, but departures do not occur every day of the week, and some days see more departures than others. The rooms available in each of the seven properties also show variation owing to sales and nonuniform block allotments. As these rooms (for these seven properties, among a list of others) are prepaid by the tour operator, one of the operator's goals is to avoid issues like the results for property A, which shows twenty-nine rooms one day and twenty-four the next,

Exhibit 1:
Sample Fourteen-Day Room and Flight Detail, Winter 2005-2006

Total seats available	218	101	14	0	96	84	203	235	169	67	0	283	197	305
Beds required	700	795	809	809	780	864	806	920	1,006	1,073	1,073	1,245	1,363	1,514
Rooms required	350	398	405	405	390	432	403	460	503	537	537	623	682	757
Property A	0	0	0	0	29	24	0	0	0	0	0	0	0	7
Property B	0	29	28	28	25	27	39	32	29	30	30	31	32	36
Property C	5	18	15	15	7	7	5	9	9	14	14	27	27	31
Property D	0	0	0	0	7	2	11	10	0	4	4	1	5	0
Property E	25	22	21	21	20	20	18	20	28	29	29	27	23	25
Property F	0	0	0	0	1	0	0	0	0	0	0	0	0	6
Property G	0	2	3	3	2	2	2	1	1	1	1	2	2	3

Exhibit 2:
Typical Booking Pace and Profit Margins for Sunquest



Note: Graph shows three sample arrival days (labeled 3rd, 10th, and 17th).

surrounded by zero rooms on preceding and successive days. As the typical guest package involves a seven-night stay at a single property, these two isolated blocks of rooms are unusable. Instead, one might prefer a pattern like that of Property E, which shows similar numbers of rooms available for seven-day stretches (or longer).

Exhibit 2 shows a graph of three sample arrival days (the 3rd, 10th, and 17th). The graph shows the booking pattern over the forty-two days prior to arrival as well as the margins during this period. The axis has been scaled to 1 for confidentiality purposes, as the actual values are not as important as the shape of the curves. The lines (solid, dashed, or dotted) represent bookings (left scale); and the points represented by diamonds, dots, and squares are margin (right scale). Demand builds quickly in the last few weeks while margins decline quickly over this same period.

The goal of the tour operator is to efficiently allocate this time-varying demand to its contracted hotels to maximize profit. In the short term, given a set of take-or-pay contracts, Sunquest's (potentially risk-averse) near-term goal is to sell or reserve prepaid (risk) rooms as quickly as possible. In the following sections, we develop a model to manage room risk, specifically to minimize unusable rooms. We present some numerical results using the tour operator's actual data and benchmark these results to methods that mimic those currently in practice.

Model Formulation

In the following section, we formulate the room-risk model as a mathematical

Exhibit 3:
Nomenclature Summary

N	Total days in the selling season
S_i	Number of risk rooms on hand at the ith night of this season
NS_i	Number of nonrisk rooms on hand at the ith night of this season
TS_i	Number of total rooms on hand at the ith night of this season, $TS_i = NS_i + S_i$
x_i	Number of rooms sold at the ith night of the season for seven nights in a row

program. For clarity of presentation, the nomenclature used is summarized in Exhibit 3. In the following formulation, we assume that the tour operator sells packages for a seven-night stay and that each such stay is at a single property. We further assume that room allotments to the tour operator for both prepaid (risk) rooms and retail (nonrisk) rooms are fixed and that demand exceeds the supply of risk rooms. While the model is developed for seven-night stays, the general framework of the model can extended to handle other stay durations, whether three nights or fourteen nights. To address nonfixed inventory allotments, the model can simply be repeatedly solved over the selling horizon as inventory levels change.

Given the goal of selling as many risk rooms as quickly as possible (or at least remove the risk from the books as soon as possible), we formulate our model with the objective of minimizing spoilage, or unsellable risk rooms. For a tour operator selling week-long vacations, on any given day, customer arrivals from the previous six days will require a room-night on the day in question. So the total number of rooms used on a given day, i, include the arrivals on that day plus stayovers from the previous six days, or $\sum_{j=i-6}^{i} x_j$. The spoiled rooms (prepaid risk rooms that go unsold) are $S_i - \sum_{j=i-6}^{i} x_j$, and total selling season spoilage is $\sum_{i=1}^{N}\left[S_i - \sum_{j=i-6}^{i} x_j\right]$.

One might assume that to minimize rooms spoiled across the whole season, the whole season would need to be formulated as one mathematical program—that is, the objective of any model would be to minimize $\sum_{i=1}^{N}\left[S_i - \sum_{j=i-6}^{i} x_j\right]$. We illustrate in *Result I* that this is not the case, but rather the sequential application of a one-day myopic formulation (that is, minimize $S_i - \sum_{j=i-6}^{i} x_j$) is equivalent to the entire-season approach. For illustration purposes, we outline in Appendix A the one-day model with the development of *Result I*.

One-Day Rolling-Window Formulation

Our objective function is to minimize the spoilage for day i by determining the number of packages to sell, x_i, for arrival day i, subject to the number of risk rooms on hand on each day and stayovers from the previous six days.

The formulation for this model is, at the ith day,

$$\text{Min } S_i - \sum_{j=i-6}^{i} x_j, \tag{1}$$

subject to

$$x_j \geq 0 \tag{2}$$

$$S_l - \sum_{j=l-6}^{i} x_j \geq 0,\ l = i, i+1, \ldots, i+6. \tag{3}$$

Here (1) is the spoiled rooms on day i (rooms available, minus stayovers minus day i's allocation x_i) and (3) makes sure that there are enough rooms over the next six days to accommodate any day i allocations.

To determine each day's allocation, this linear program would be sequentially run for each day of the season starting at the first day, with subsequent days using the decisions from previous days as inputs to determine stayovers.

The above formulation looks at risk rooms in isolation. To be more realistic, a model should take into account the fact that the tour operator often has additional rooms available at a particular property on a spot or nonrisk basis (NS_i). These nonrisk rooms have the potential to add to the available supply of rooms such that more risk rooms can be utilized. This is true because consumers do not care (or even know) whether they are staying in a risk or nonrisk room as long as they are getting their seven nights at the property. The following develops a nonlinear formulation for the allocation of risk and nonrisk rooms jointly.

Inclusion of Nonrisk Rooms

The inclusion of nonrisk rooms as available rooms, while conceptually not that different, changes the formulation enough that it is no longer linear, nor is the one-day myopic formulation optimal. Below we develop the mathematical program for minimizing spoilage across the entire selling season, using both at-risk and nonrisk rooms. In Appendix B, *Result II* illustrates that the single-day rolling-window formulation is not necessarily globally optimal as in the risk-room-only case.

This formulation is similar to the case of risk-only rooms, except that the total rooms on hand, labeled TS_i, is now the sum of risk rooms and nonrisk rooms for each property.

The total spoilage becomes $\sum_{i=1}^{N}\left[\max\left(S_i - \sum_{j=i-6}^{i} x_j, 0\right)\right]$.

The whole season model, selling seven-day packages in this case is as follows:

$$\min \sum_{i=1}^{N}\left[\max\ S_i - \sum_{j=i-6}^{i} x_j, 0\right)\Bigg]. \quad (4)$$

subject to

$$x_i \geq 0,\ i = 1, 2, \ldots, N$$

$$TS_i - \sum_{j=i-6}^{i} x_j \geq 0,\ i = 1, 2, \ldots, N,$$

where TS_i is the daily sum of total rooms available, both risk and nonrisk.

In the following section, we illustrate application of the above two formulations to the seven hotel properties during the winter 2005-2006 season.

Numerical Example

Using data available from Sunquest Vacations for a representative winter destination, we outline the impacts of optimal risk-room allocation. We use data from a 151-day selling horizon for a high-volume destination for all seven properties that Sunquest engaged in take-or-pay contracts for room acquisition. As Exhibit 4 shows, these seven properties have different degrees of risk. Property A has more than 10,000 rooms committed to inventory, whereas property G has only 258.

The numerical results are summarized in Exhibit 4. The table includes three model formulations as well as a set of benchmark spoilage results, as follows: the one-day model using only risk-based rooms as available supply, a nonlinear version of this model using nonrisk rooms to augment supply, and a model using nonrisk and risk rooms that models the whole season simultaneously.

As a benchmark to evaluate the impact of the room-allocation approaches, current practice is replicated (labeled "benchmark spoilage" in Exhibit 4). In an effort to provide a benchmark, we created a proxy for current practice. Current practice has no scientific approach to the allocation of risk rooms but, rather, attempts to sell risk rooms as fast as possible as requests come in. For example, whenever a reservation is requested for seven nights at property A, this request is fulfilled with risk rooms until risk rooms are depleted (or no saleable combinations exist). To replicate this

Exhibit 4:
Sample Results for Seven Representative Properties

	Properties						
	A	*B*	*C*	*D*	*E*	*F*	*G*
Risk supply	10,508	5,518	3,204	2,539	3,999	1,055	258
Model: Spoilage %							
Risk only	8.3	8.5	5.0	22.5	3.2	20.4	7.8
Risk and Nonrisk	1.4	3.8	1.7	16.2	3.2	3.5	7.8
Risk and nonrisk season model	0.0	0.0	0.0	8.1	1.9	2.8	3.1
Benchmark spoilage	29.5	30.0	26.6	32.9	23.1	39.4	27.0

Exhibit 5:
Nonrisk Room Availability and Usage

	Properties						
	A	*B*	*C*	*D*	*E*	*F*	*G*
Risk supply	10,508	5,518	3,204	2,539	3,999	1,055	258
Risk sold	10,508	5,518	3,204	2,338	3,925	1,025	238
Nonrisk supply	9,402	4,034	2,505	1,052	183	1,578	30
Nonrisk used	2,862	2,546	1,409	581	150	872	30

approach, we randomly selected arrival days and then minimized spoilage on that arrival day (using the risk-rooms-only formulation), and we sampled arrival days without replacement until the entire season of arrival days had been allocated. We performed ten replications, resulting in the averages presented in Exhibit 4. As one can see, considerable spoilage results from the nonsystematic allocation of risk rooms as today's decisions are increasingly limited by past decisions when these decisions are made nonsequentially.

As shown in Exhibit 4, the inclusion of nonrisk rooms into the allocation of risk rooms greatly reduces spoilage. While the availability of nonrisk rooms is not within the control of the tour operator (as other firms and the hotel itself are free to sell these rooms as well), as long as nonrisk room availability approaches that of risk rooms, then the uncertainty in supply of nonrisk rooms should not significantly alter spoilage. Exhibit 5 summarizes this principle for the seven properties evaluated, showing the availability and usage of nonrisk rooms in the allocation of the risk rooms for the selling season that we studied. The availability of nonrisk rooms is variable for the seven properties in question, but fortunately, with the exception of property E, properties in which Sunquest has large risk positions also have considerable spot or nonrisk availability. We emphasize that nonrisk room availability is subject to prior sale and continually changes, resulting in the probable need to repeatedly solve the mathematical program over time as (nonrisk) inventory becomes depleted.

For illustration purposes, Exhibit 6 shows a sample set of available rooms, allocations, and the resulting spoilage for a twenty-one-day window from property E, with the tour operator selling seven-night packages. The *Rooms* row in Exhibit 6 contains the number of available risk rooms for each of the twenty-one days. The *Optimal sales* row, the decisions made by the model, contains the optimal sales to occur on each of these twenty-one days,

Exhibit 6:
Sample Spoilage

	Day																				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Rooms	23	22	19	22	22	24	24	30	25	22	21	21	20	20	18	20	28	29	29	27	23
Optimal sales	19	0	0	3	0	2	0	17	0	0	2	0	1	0	15	2	6	0	0	0	0
Spoilage	4	3	0	0	0	0	0	8	3	0	0	0	0	0	0	0	2	5	5	4	0

with *Spoilage* representing rooms that cannot be sold. On day 1 only nineteen packages can be sold, because day 3 only has nineteen rooms available, restricting sales of seven-day packages. This results in spoilage of four rooms on day 1 and three rooms on day 2. An additional three packages can be sold on day 4, resulting in no spoilage until day 8, when only seventeen additional packages can be sold, even though there are thirty rooms available on day 17. As on day 1, day 17 sales are limited by the rooms available on each of the next six days, complicated by the number of stayovers from earlier days. This means spoilage of eight rooms on day 8, calculated as thirty rooms minus seventeen sales minus five stayovers (from days 4 and 6).

Conclusion

This article has presented a brief introduction into aspects of the revenue management considerations of a tour operator. Specifically, we examined the allocation of prepaid rooms, which we call risk rooms, with the objective of using as many of these rooms as possible (to minimize spoilage). We demonstrated that if an operator wishes to focus only on the risk rooms, a relatively straightforward model can be repetitively applied, resulting in significant improvements in room utilization. Our calculation showed a reduction in spoiled rooms exceeding 25 percent. The inclusion of rooms that are acquired on a nonrisk basis adds flexibility to the allocation of risk rooms and provides a concomitant reduction in wasted rooms by a further 2 percent. The complication relating to the nonrisk rooms is that their availability is not within the tour operator's control. As rooms are sold through other channels, the mathematical model relating to those rooms must be repeatedly solved with new data as the nonrisk inventory becomes depleted. The lack of control on nonrisk rooms owing to uncertain demand serves as a limitation of the model formulation, but the reapplication of the developed model similar to the reapplication of traditional revenue management models (Talluri and van Ryzin, 2004) should limit the negative effects of this uncertainty. As the tour operator reduces its spoiled rooms (assuming demand is constant, sales would probably increase as spoilage reduction reduces room cost, making the firm more price competitive), it directly increases its profit, particularly since it can restrict its purchases of nonrisk rooms on the spot or retail market.

Appendix A

Result I—Equivalence of One-Day and Seasonwide Formulations

Total spoilage of a one-day rolling-window model is equal to that of the whole-season model. Consider a simple example of a one-day rolling-window model for a three-day window, selling two-night hotel stays. The two decisions x_i, $I = 1, 2$ are the number of rooms sold on the ith day for two days in a row. The objective function is to minimize daily spoilage.

Assume that $\bar{x}_1$, $\bar{x}_2$ are the optimal solution for this model with total spoilage $\sum_{i=1}^{3} S_i - 2(\bar{x}_1 + \bar{x}_2)$.

The formulation is as follows:

$$\text{Min } S_1 - x_1, \qquad \text{(A1-1)}$$

subject to

$$x_1 \geq 0$$
$$S_i - x_1 \geq 0, \text{ where } i = 1,2, \qquad \text{(A1-2)}$$

and

$$\text{Min } S_2 - \bar{x}_1 - x_2$$

subject to

$$x_2 \geq 0$$
$$S_2 - x_1 - x_2 \geq 0$$
$$S_3 - x_2 \geq 0.$$

By observation, we get $\bar{x}_1 = \min(S_1, S_2)$ and $\bar{x}_2 = \min(S_2 - \bar{x}_1, S_3)$.

If $S_1 \leq S_2$,

1. If $S_2 - S_1 \leq S_3$, $\bar{x}_1 = S_1$, $\bar{x}_2 = S_2 - S_1$, total spoilage $= \sum_{i=1}^{3} [S_i - 2S_2] = S_3 - S_2 + S_1$.
2. If $S_2 - S_1 > S_3$, $\bar{x}_1 = S_1$, $\bar{x}_2 = S_3$, total spoilage $= \sum_{i=1}^{3} [S_i - 2S_3 - 2S_1] = S_3 - S_2 - S_1]$.
3. If $S_1 > S_2$, i.e. $S_2 - S_1 < 0$, then $\bar{x}_1 = S_2$, $\bar{x}_2 = 0$, total spoilage $= \sum_{i=1}^{3} [S_i - 2S_2] = S_3 - S_2 + S_1$.

In summary, the results are

- If S2 – S1 ≤ S3, total spoilage = S3 – S2 + S1,
- If S2 – S1 > S3, total spoilage = S2 – S1 – S3.

For a whole season model with a three day window, the two decision variables x_i, $i = 1, 2$ are the same with the objective function minimizing total spoilage for the entire three day window. Assume that x_1^*, x_2^* are the optimal solution for this model. Total spoilage $= \sum_{i=1}^{3} S_i - 2(x_1^* + x_2^*)$.

Therefore, the formulation is as follows:

$$\text{Min} \sum_{i=1}^{3} S_i - 2(x_1 + x_2), \qquad \text{(A1-3)}$$

subject to

$$x_1, x_2 \geq 0$$
$$S_1 - x_1 \geq 0$$
$$S_2 - x_1 - x_2 \geq 0$$
$$S_3 - x_2 \geq 0.$$

From the formulation, for a given optimal $x_1(x_1^*)$, we can get

$$\text{Min} \sum_{i=1}^{3} S_i - 2(x_1^* + x_2), \qquad \text{(A1-4)}$$

subject to

$$x_2 \geq 0$$
$$S_2 - x_1^* - x_2 \geq 0$$
$$S_3 - x_2 \geq 0.$$

Then, by observation, $x_2^* = \min(S_3, S_2 - x_1^*)$.

Similarly, for optimal x_2^*,

$$\text{Min} \sum_{i=1}^{3} S_i - 2(x_1 + x_2^*), \qquad \text{(A1-5)}$$

subject to

$$x_1 \geq 0$$
$$S_1 - x_1 \geq 0$$
$$S_2 - x_1 - x_2^* \geq 0,$$

then, by observation, $x_2^* = \min(S_1, S_2 - x_2^*)$.

Therefore,

1. If $S_2 - x_1^* \leq S_3$, that is, $x_1^* \geq S_2 - S_3$, then x_2^* $S_1 \geq S_2 - x_1^*$, that is, $x_2^* + x_1^* = S_2$. So total spoilage $= \sum_{i=1}^{3} S_i - 2S_2 = S_1 - S_2 + S_3$. Also we know that $x_1^* = \min(S_1, S_2 - x_2^*)$, so $S_1 \geq x_1^* \geq S_2 - S_3$, that is, $S_2 - S_1 \leq S_3$. Thus, when $S_2 - S_1 \leq S_3$, total spoilage $= \sum_{i=1}^{3} S_i - 2S_2 = S_1 - S_2 + S_3$.

2. If $S_2 - x_1^* > S_3$, then $x_2^* = S_3$, $x_1^* = \min(S_1, S_2 - S_3)$. So if $S_1 < S_2 - S_3$, then $x_1^* = S_1$, and total spoilage $= \sum_{i=1}^{3} S_i - 2S_3 - 2S_1 = S_2 - S_1 - S_3$. If $S_1 \geq S_2 - S_3$, then $x_1^* = S_2 - S_3$, and total spoilage $= \sum_{i=1}^{3} S_i - 2S_2 = S_1 - S_2 + S_3$.

In summary the results are

- If $S_2 - S_1 \leq S_3$, total spoilage $= \sum_{i=1}^{3} S_i - 2S_2 = S_1 - S_2 + S_3$.
- If $S_2 - S_1 > S_3$, total spoilage $= \sum_{i=1}^{3} S_i - 2S_3 - 2S_1 = S_2 - S_1 - S_3$.

Which is the same as the one-day rolling-window formulation.

Appendix B

Result II—Nonequivalence of One-Day and Seasonwide Formulations with Inclusion of Nonrisk Rooms

In this case, the one-day rolling-window model does not necessarily have the same spoilage as the whole-season model has. Similar to *Result I*, we illustrate a simple example with a one-day rolling-window model for a three-day season, selling two-day packages.

The decision variables x_i, $i = 1, 2$, the number of rooms sold on day i for two days in a row, with the objective to minimize each day's spoilage. Assume that $\bar{x}_1$, $\bar{x}_2$ are the optimal solution for this model with total spoilage $\max(0, S_1 - \bar{x}_1) + \max(0, S_2 - \bar{x}_1 - \bar{x}_2) + \max(0, S_3\ \bar{x}_2)$.

The formulation is as follows:

$$\text{Min } \max(0, S_1 - x_1), \qquad \text{(A2-1)}$$

subject to

$$x_1 \geq 0$$
$$TS_i - x_1 \geq 0, \text{ where } i = 1, 2$$

and

$$\text{Min } \max(0, S_2 - \bar{x}_1 - x_2) \qquad \text{(A2-2)}$$

subject to

$$x_2 \geq 0$$
$$TS_2 - \bar{x}_1 - x_2 \geq 0$$
$$TS_3 - x_2 \geq 0.$$

By observation, we get $\bar{x}_1 = \min(TS_1, TS_2)$ and $\bar{x}_2 = \min(TS_2 - \bar{x}_1, TS_3)$.

If $TS_1 \leq TS_2$, then

1. If $TS_2 - TS_1 \leq TS_3$, x–1 $= TS_1$, $\bar{x}_2 = TS_2 - TS_1$.
   Total spoilage $= \max(0, S_1 - x_1) + \max(0, S_2 - x_1 - x_2) + \max(0, S_3 - x_2)$
   $= 0 + 0 + \max(0, S_3 - TS_2 + TS_1)$, since $S_i \leq TS_i$ for all i.
   = max(0, S2 –TS2 +TS1).
2. If $TS_2 - TS_1 > TS_3$, $\bar{x}_1 = TS_1$, $\bar{x}_2 = TS_2$.
   Total spoilage = max(0, S1 – x1) + $\max(0, S_2 - x_1 - x_2) + \max(0, S_3 - x_2)$
   $= 0 + \max(0, S_2 - TS_3 - TS_1)$
   If $TS_1 > TS_2$, that is, $TS_2 - TS_1 < 0 < TS_3$, then $\bar{x}_1$ =TS1, $\bar{x}_2 = 0$.
   Total spoilage is then $\max(0, S_1 - x_1) + \max(0, S_2 - x_1 - x_2) + \max(0, S_3 - x_2)$
   $= 0 + 0 + \max(0, S_3 - 0) = S_3$.

In summary, the results are

- If $TS_1 \leq TS_2$, then: if $TS_2 - TS_1 \leq TS_3$, then total spoilage $= \max(0, S_3 - TS_2 + TS_1)$, if $TS_2 - TS_1 > TS_3$, then total spoilage $= \max(0, S_2 - TS_3 - TS_3)$.
- If $TS_1 > TS_2$, total spoilage $= S_3$.

Similarly, for a model of the entire season for a three-day window, with the objective of minimizing total season spoilage, assume that x_1^*, x_2^* are the optimal solution for this model. Total spoilage $= \max(0, S_1 - x_1^*) + \max(0, S_2 - x_1^* - x_2^*) + \max(0, S_3 - x_2^*)$.

The formulation is as follows:

$$\text{Min } [\max(0, S_1 - x_1) + \max(0, S_2 - x_1 - x_2) + \max(0, S_3 - x_2) \qquad \text{(A2-3)}$$

subject to

$$x_1, x_2 \geq 0$$
$$TS_1 - x_1 \geq 0$$
$$TS_2 - x_1 - x_2 \geq 0$$
$$TS_3 - x_2 \geq 0.$$

From the formulation, we can get

$$\text{Min } [\max(0, S_1 - x_1^*) + \max(0, S_2 - x_1^* - x_2) + \max(0, S_3 - x_2) \quad \text{(A2-4)}$$

subject to

$$x_2 \geq 0$$
$$TS_2 - x_1^* - x_2 \geq 0$$
$$TS_3 - x_2 \geq 0.$$

Then by observation, we get $x_2^* = \min(TS_3, TS_2 - x_1^*)$.

$$\text{Min } [\max(0, S_1 - x_1) + \max(0, S_2 - x_1 - x_2^*) + \max(0, S_3 - x_2^*) \quad \text{(A2-5)}$$

subject to

$$x_1 \geq 0$$
$$TS_1 - x_1 \geq 0$$
$$TS_2 - x_1 - x_2^* \geq 0.$$

Then by observation, we get $x_1^* = \min(TS_1, TS_2 - x_2^*)$.

Therefore, if $TS_2 - x_1^* \leq TS_3$, then $x_2^* = TS_2 - x_1^*$, total spoilage $= \max(0, S_1 - x_1^*) + \max(0, S_2 - x_1^* - x_2^*) + \max(0, S_3 - x_2^*) = \max(0, S_1 - x_1^*) + \max(0, S_3 - x_2^*)$.

Also, $x_1^* = \min(TS_1, TS_2 - x_2^*)$ so $TS_1 \geq x_2^* \geq TS_2 - TS_3$ that is, $TS_2 - TS_1 \leq TS_3$.

Thus, if $TS_1 \leq TS_2$ and $TS_1 > S_1 > TS_2 - TS_3$, then when $S_1 = x_1$, the total spoilage $= \max(0, S_1 - x_1) + \max(0, S_3 - x_2) = \max(0, S_3 - TS_2 + S_1)$, so the optimal spoilage $= \max(0, S_1 - x_1^*) + \max(0, S_2 - x_1^* - x_2^*) + \max(0, S_3 - x_2^*) \leq \max(0, S_3 - TS_2 + S_1)$. So if $S_3 - TS_2 + S_1 > 0$, then the total spoilage $\leq S_3 - TS_2 + S_1$; but recall that under the same conditions, the total spoilage for one-day rolling window $= \max(S_3 - TS_2 + TS_1, 0) = S_3 - TS_2 + TS_1 > S_3 - TS_2 + S_1$.

Therefore, we have found a situation where these two models have different spoilage while including the nonrisk rooms as supply data.

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